Belief Contraction and Revision over DL-Lite TBoxes

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Abstract

In this paper, we study the operations of contraction and revision for removing and incorporating axioms over DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$ TBoxes (Calvanese et al. 2007). The operations are defined in the manner of AGM model-based contraction and revision (Grove 1988; Katsuno and Mendelzon 1992) and are based on a newly defined semantics called type semantics. We show that, as an alternative to description logic (DL) semantics, type semantics is better suited for defining contraction and revision for DL-Lite. We provide characterisations for the defined operations through AGM style rationality postulates. We also provide a quadratic time non-deterministic algorithm for instantiating one of the contraction operations.

Introduction

Ontology, together with its underlying logical formalism, description logics (DLs) (Baader et al. 2003), is becoming into a prominent knowledge sharing technique in e-Health, bio-informatics and the semantic web. Although DLs are not designed to represent evolving knowledge, the engineering and maintenance of ontologies are a dynamic process. As a result, extensive attention has been paid to the problem of ontology changes. Especially, many have proposed for adapting classic belief change operations to DLs, e.g. (Qi and Du 2009).

Belief change deals with how an agent maintains its set of beliefs in the face of new information. The dominant approach is the so called AGM framework (Alchourrón and Makinson 1985; Gärdenfors and Rott 1995). Essentially, the framework assumes classical propositional logic which is captured by the standard Tarskian consequence operator Cn. Two change operations are mainly studied which are *contraction* for removing a formula from the belief set and *revision* for incorporating a formula into the belief set. The main strategies for studying the change operations are to articulate principles called *rationality postulates* that captures the intuitions behind rational belief change and to specify explicit change mechanisms called *construction methods* for the operation. It is commonly accepted that the AGM framework provides the best set of postulates and constructions.

A crucial task in belief change is to derive the so called *representation theorem* that assures the soundness and completeness of a construction method with respect to a set of rationality postulates. Intuitively, it says the construction method is rational as the constructed operation satisfies a set of intuitively plausible postulates and it is also comprehensive as all change operators satisfying the postulates can be constructed through this method. We often say the postulates specified in a representation theorem *characterise* the construction. As the hall mark of AGM framework, all its major constructions are equipped with representation theorems.

Even though the AGM postulates can be recast so as to be applicable to non-classical logics, it is non-trivial to adapt the construction methods to non-classic logics and to derive their representation theorems. For DLs, this problem is challenging for at least two reasons: (1) We are dealing with contraction and revision for fragments of first order logic in which expressivity can be an issue, and (2) For revision, in addition to inconsistency we also have to deal with incoherence. As far as we know, existing works on adapting AGM contraction and revision to DL have failed to give such characterisations and to switch properly from inconsistency handling to incoherence handling for revision. In this paper we address the above open problems for DL-Lite (Calvanese et al. 2007) which underlies the OWL 2 QL profile of OWL 2 and gains its popularity through its efficient query answering. Specifically, we make the following contributions:

- We first introduce a new characterisation for the semantics of DL-Lite, which is based on a syntactic notion called *types*. Informally, a type is set of basic concepts and roles which is alike an classic interpretation represented by atoms assigned true. A type is a *type model* of a TBox $\mathcal{T}$ if it satisfies each axiom of $\mathcal{T}$. The type semantics is equivalent to the semantics of DL-Lite w.r.t. all major TBox reasoning tasks. The advantage of type semantics over DL semantics is that (1) For a set of DL models M, there may not exist a DL-Lite TBox whose set of models is M. Such inexpressibility also appear in type semantics but it is much easier to manage compare to the DL case. (2) Type semantics is more compact than DL semantics such that it is feasible to develop tractable algorithms for type model-based contraction and revision and (3) Type semantics resembles classic semantics which make it more suitable for defining AGM style model-based contraction and revision.

- We then adapt the full set of AGM contraction postulates and some of their alternatives to DL-Lite. Based on type semantics, we define two contraction operations for DL-Lite TBoxes. We show representation theorems establishing the equivalence between the constructions and the adapted postulates. We also provide a quadratic time algorithm to compute a restricted form of the contraction operation.
- Finally, we adapt the basic AGM revision postulates to DL-Lite. Crucial to the adaptation is the switch from inconsistency handling to incoherence handling. Based on type semantics, we define a type model-based revision for DL-Lite TBoxes and obtain its representation theorem.

A longer version of the paper with proofs can be found at zhiqiangzhuang.com/Home_files/DLcontraction.pdf.

Preliminary

AGM Model-Based Contraction and Revision

In this section, we introduce the model-theoretic approach in constructing AGM contraction and revision (Grove 1988; Katsuno and Mendelzon 1992). We present the constructions in the style of (Katsuno and Mendelzon 1992).

The universal set of interpretations is denoted as Ω. The models of a set of formulas S (a formula ϕ) is denoted as $CM(S)$ (reps., $CM(\phi)$). $\mathcal{T}_{prop}$ is a function such that given a set of interpretations M, $\mathcal{T}_{prop}(M)$ is a logically closed set of formulas whose set of models is M. A *preorder* $\preceq$ is a reflexive and transitive binary relation over Ω. The strict relation $\prec$ is defined as $\mu \prec \nu$ iff $\mu \preceq \nu$ and $\nu \not\preceq \mu$. A preorder is *total* if for every pair of interpretations $\mu, \nu \in \Omega$, either $\mu \preceq \nu$ or $\nu \preceq \mu$. Let M be a set of interpretations, $min(M, \preceq)$ represents the minimal elements of M such that $min(M, \preceq) = \{\mu \in M :$ there is no $\nu \in M$ such that $\nu \prec \mu\}$. Each belief set K is assigned a preorder which represents a measure of closeness between models of K and an interpretation such that $\mu \preceq \nu$ means μ is at least as close to models of K as ν is. Intuitively, models of K are the closest to themselves. Preorders with this property are called *faithful*. Formally, a preorder $\preceq$ is faithful with respect to K iff $min(\Omega, \preceq) = CM(K)$.

A function $\dot{-}$ is an AGM model-based contraction function for K iff $K \dot{-} \phi = \mathcal{T}_{prop}(CM(K) \cup min(CM(\neg\phi), \preceq))$ for all formulas ϕ, where $\preceq$ is a faithful preorder. The corresponding representation theorem shows that the contraction is characterised by the following postulates:

$(K\dot{-}1)$	$K\dot{-}\phi = Cn(K\dot{-}\phi)$
$(K\dot{-}2)$	$K\dot{-}\phi \subseteq K$
$(K\dot{-}3)$	If $\phi \notin K$, then $K\dot{-}\phi = K$
$(K\dot{-}4)$	If $\not\vdash \phi$, then $\phi \notin K\dot{-}\phi$
$(K\dot{-}5)$	$K \subseteq (K\dot{-}\phi) + \phi$
$(K\dot{-}6)$	If $\phi \equiv \psi$, then $K\dot{-}\phi = K\dot{-}\psi$
$(K\dot{-}7)$	$K\dot{-}\phi \cap K\dot{-}\psi \subseteq K\dot{-}\phi \wedge \psi$
$(K\dot{-}8)$	If $\phi \notin K\dot{-}\phi \wedge \psi$ then $K\dot{-}\phi \wedge \psi \subseteq K\dot{-}\phi$

In the AGM tradition, $(K\dot{-}1)$–$(K\dot{-}6)$ are referred to as the *basic postulates* and $(K\dot{-}7)$ and $(K\dot{-}8)$ the *supplementary postulates*. The postulates for revision are classified in the same manner. Notice that $+$ is the *expansion* operation such that $K + \phi = Cn(K \cup \{\phi\})$.

Paramount to all change operations is the *principle of minimal change* (Gärdenfors 1988). $(K\dot{-}5)$ (i.e., Recovery) is the main postulate for formalising this principle for contraction. It requires the information loss during contraction to be minimal such that the original belief set can be recovered by expanding the contracting formula. The postulate is argued in (Hansson 1991) to be an emerging property rather than a fundamental postulate for contraction. Other than the construction of the contraction, its satisfaction relies also on properties of the underlying logic (Ribeiro and Wassermann 2009). In particular most of the DLs including DL-Lite are incompatible with Recovery. Several well known alternatives have been proposed such as Relevance and Core-retainment (Hansson 1991). Both postulates can replace Recovery in characterising AGM contraction and are more fundamental in the sense that they are applicable to a wider classes of logics (Hansson and Wassermann 2002; Ribeiro and Wassermann 2009). Therefore, when constructing contraction under non-classic logics, dissatisfaction of Recovery is not a defect of the constructed contraction as long as it satisfies some other more fundamental alternatives. It then comes down to identifying the appropriate alternatives. As noticed in (Fermé, Krevneris, and Reis 2008), Recovery can also be replaced by the following postulate of Disjunctive elimination in characterising AGM contraction:

If $\psi \in K$ and $\phi \vee \psi \in K\dot{-}\phi$ then $\psi \in K\dot{-}\phi$.

Both Relevance and Core-retainment capture the minimal change principle by stating conditions for removing a formula during a contraction. That is only the formulas that are responsible for the entailment of the contracting formula should be removed. In contrast, Disjunctive elimination captures the principle by stating the condition for retaining a formula during a contraction. That is if a formula is in the original belief set and its disjunction with the contacting formula is retained during the contraction then the formula is retained.

In the presence of the basic postulates, $(K\dot{-}7)$ (i.e., Conjunctive overlap) is equivalent to Conjunctive trisection (Rott 1992; Hansson 1993):

$(K\dot{-}ct)$ If $\phi \in K\dot{-}(\phi \wedge \psi)$ then $\phi \in K\dot{-}(\phi \wedge \psi \wedge \delta)$

Roughly speaking, Conjunctive trisection requires that the better one (i.e., ϕ) of a pair (i.e., ϕ, ψ) is not the worst one when a third one (i.e., δ) is added to the pair.

A function $*$ is a AGM model-based revision function for a belief set K iff $K * \phi = \mathcal{T}_{prop}(min(CM(\phi), \preceq))$ for all formulas ϕ, where $\preceq$ is a faithful total preorder. The corresponding representation theorem shows that the revision is characterised by the following postulates:

$(K * 1)$	$K * \phi = Cn(K * \phi)$
$(K * 2)$	$\phi \in K * \phi$
$(K * 3)$	$K * \phi \subseteq K + \phi$
$(K * 4)$	If $K \cup \{\phi\}$ is consistent, then $K + \phi \subseteq K * \phi$
$(K * 5)$	If ϕ is consistent then $K * \phi$ is consistent

$(K*6)$	If $\phi \equiv \psi$, then $K * \phi = K * \psi$
$(K*7)$	$K * \phi \wedge \psi \subseteq (K * \phi) + \psi$
$(K*8)$	If $(K * \phi) \cup \{\phi\}$ is consistent, then $(K * \phi) + \psi \subseteq K * \phi \wedge \psi$

DL-Lite

In this section we introduce the family of DL-Lite languages. The core of the family is DL-Lite$_{core}$ which has the following syntax:

$$B \rightarrow A \mid \exists R \qquad R \rightarrow P \mid P^-$$
$$C \rightarrow B \mid \neg B \qquad E \rightarrow R \mid \neg R$$

where A denotes an *atomic concept*, P an *atomic role*, P^- the *inverse* of the atomic role P. B denotes a *basic concept* which can be either an atomic concept or an unqualified existential quantification on basic role. C denotes a *general concept* which can be either an basic concept or its negation. E denotes a *general role* which can be either an atomic role or its negation. We also include $\bot$ (denoting the empty set) and $\top$ (denoting the whole domain). We use $\mathcal{B}$ to represent the universal set of basic concepts and $\mathcal{R}$ as the universal set of atomic roles and their inverses. For an inverse role $R = P^-$, we write R^- meaning P for the convenience of presentation. In this paper, we assume $\mathcal{B}$ and $\mathcal{R}$ to be finite.

A DL-Lite$_{core}$ knowledge base consists of a TBox and an ABox. A TBox is a finite set of *concept inclusion axioms* of the form $B \sqsubseteq C$, $B \sqsubseteq \bot$, and $\top \sqsubseteq C$. That is only basic concept or $\top$ can appear on the left-hand side of a concept inclusion. An ABox is a finite set of *assertions* of the form $A(a)$ or $P(a, b)$.

There are two major extensions of DL-Lite$_{core}$, namely DL-Lite$_{\mathcal{R}}$ and DL-Lite$_{\mathcal{F}}$. DL-Lite$_{\mathcal{R}}$ extends DL-Lite$_{core}$ with *role inclusion axioms* of the form $R \sqsubseteq E$. That is only basic roles can appear on the left-hand side of a role inclusion. DL-Lite$_{\mathcal{F}}$ extends DL-Lite$_{core}$ with assertions of the form $(\texttt{funct}\ R)$ which specifies functionality on basic roles. Frequently, TBox axioms will be denoted by lower case Greek letters $(\phi, \psi, \ldots)$.

The semantics of a DL-Lite language is given in terms of interpretations. An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a nonempty domain $\Delta^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$ that assigns to each atomic concept A a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$, and to each atomic role P a binary relation $P^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$, and to each individual name a an element $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$. The interpretation function is extended to general concept, general roles, and special symbols as follows:

$$\bot^{\mathcal{I}} = \emptyset$$
$$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$$
$$(P^-)^{\mathcal{I}} = \{(o_2, o_1) \mid (o_1, o_2) \in P^{\mathcal{I}}\}$$
$$(\exists R)^{\mathcal{I}} = \{o \,:\, \exists o^{'}.(o, o^{'}) \in R^{\mathcal{I}}\}$$
$$(\neg B)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus B^{\mathcal{I}}$$
$$(\neg R)^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus R^{\mathcal{I}}$$

An interpretation $\mathcal{I}$ satisfies a concept inclusion $B \sqsubseteq C$ if $B^{\mathcal{I}} \subseteq C^{\mathcal{I}}$, a role inclusion $R \sqsubseteq E$ if $R^{\mathcal{I}} \subseteq E^{\mathcal{I}}$, a concept assertion $A(a)$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$, a role assertion $P(a, b)$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}}$, and a functionality assertion $(\texttt{funct}\ R)$ if $(o, o_1) \in R^{\mathcal{I}}$ and $(o, o_1) \in R^{\mathcal{I}}$ implies $o_1 = o_2$. $\mathcal{I}$ satisfies a TBox $\mathcal{T}$ (or ABox $\mathcal{A}$) if $\mathcal{I}$ satisfies each axiom in $\mathcal{T}$ (resp., each assertion in $\mathcal{A}$). $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ (or a TBox axiom ϕ) denoted as $\mathcal{I} \models \mathcal{T}$ (resp., $\mathcal{I} \models \phi$) if it satisfies $\mathcal{T}$ (resp., ϕ). A TBox or an axiom is *consistent* if it has at least one model. A TBox $\mathcal{T}$ logically implies an axiom ϕ, written $\mathcal{T} \models \phi$, if all models of $\mathcal{T}$ are also models of ϕ. Two TBox axioms ϕ and ψ are logically equivalent, written $\phi \equiv \psi$, if they have identical set of models. The logical closure of a TBox $\mathcal{T}$, denoted as $cl(\mathcal{T})$, is the set of all TBox axioms ϕ such that $\mathcal{T} \models \phi$. In the upcoming sections all TBoxes are assumed to be closed. We use $\vdash \phi$ to denote that ϕ is an tautology such as $A \sqsubseteq A$. We use $\{\top \sqsubseteq \bot\}$ to denote the inconsistent TBox.

A basic concept B is *satisfiable* with respect to a TBox $\mathcal{T}$ if there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $B^{\mathcal{I}}$ is non-empty, and B is *unsatisfiable* if $B^{\mathcal{I}} = \emptyset$ for every model $\mathcal{I}$ of $\mathcal{T}$. It is easy to see that B is unsatisfiable with respect to a TBox $\mathcal{T}$ iff $B \sqsubseteq \bot \in cl(\mathcal{T})$. A TBox is *coherent* if all basic concepts are satisfiable and *incoherent* otherwise. Notice that, often in DL literatures, coherence comes with the absence of unsatisfiable atomic concepts. Since, in DL-Lite, unsatisfiable non-atomic concepts like $\exists R$ are also unexpected we use the stricter condition for coherence.

In general, TBox axioms are logically connected. One exception is the functionality assertions. For DL-Lite$_{\mathcal{F}}$, if ABox is not considered, a functionality assertion do not imply and is not implied by any other axioms, thus its removal and addition is nothing but set operations. For this reason, we only consider contraction and revision over DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$ TBoxes.

We allow contraction and revision of not only single TBox axioms but also conjunctions of them. For TBox axioms ϕ and ψ, their conjunction is denoted as $\phi \wedge \psi$. An interpretation $\mathcal{I}$ satisfies $\phi \wedge \psi$ if it satisfies both ϕ and ψ.

One unique feature of DL-Lite is that any concept that can be formed is finite in length. Most DLs that allow qualified existential or universal quantification do not have such finiteness property. For instance, in $\mathcal{EL}$, a concept can be formed through an unbounded nesting of existential quantifiers such as $\exists R \exists R \cdots \exists S$ thus the length of the concept is infinite. The finiteness property is crucial in defining the alternative semantics for DL-Lite TBoxes in the next section.

Semantic Characterisation for DL-Lite TBox

In this section we study type sematnics which is built on top of the classic semantics for propositional logic with a few extras to handle the non-classic inferences of DL-Lite TBoxes. Central to the semantics is the notion of types which is introduced in (Kontchakov, Wolter, and Zakharyaschev 2008). We first present the type semantics for DL-Lite$_{core}$ and later for DL-Lite$_{\mathcal{R}}$. Due to the allowance of role inclusions, the type semantics for DL-Lite$_{\mathcal{R}}$ extends that of DL-Lite$_{core}$ with role names.

A type $\tau \subseteq \mathcal{B}$ for DL-Lite$_{core}$ is a possibly empty set of basic concepts. The universal set of types for DL-Lite$_{core}$ is denoted as Ω_{core}. By considering basic concepts as propositional atoms and concept inclusions $B \sqsubseteq C$ as propositional formulas $\neg B \vee C$, a type is simply a classic interpretation (represented by atoms interpreted as true) of the formulas. Given a DL-Lite$_{core}$ TBox $\mathcal{T}$, we use $CM(\mathcal{T})$ to denote the set of types that consists of classic models of the corresponding propositional formulas of $\mathcal{T}$. Our first thought is to use classic models as the semantic characterisation for a TBox. Under an appropriate semantics, any model of the logically stronger formula should be one of the weaker. To this respect using classic models is inappropriate. For instance, although $\exists R \sqsubseteq \bot$ implies $\exists R^- \sqsubseteq \bot$, the type $\{\exists R^-\}$ which is a classic model of $\exists R \sqsubseteq \bot$ is not one of $\exists R^- \sqsubseteq \bot$.

The reason for the counter-intuitiveness is that the inference from $\exists R \sqsubseteq \bot$ to $\exists R^- \sqsubseteq \bot$ is non-classic. Special treatment is thus needed for axioms of the form $\exists R \sqsubseteq \bot$. We give below the conditions under which a type is a model of a DL-Lite$_{core}$ TBox. To distinguish from classic models, these models will be called *type models* for DL-Lite$_{core}$.

Definition 1. *A type τ is a type model of a DL-Lite$_{core}$ TBox $\mathcal{T}$ iff*

1. $\tau \in CM(\mathcal{T})$.
2. *if* $\exists R \sqsubseteq \bot \in cl(\mathcal{T})$ *then* $\exists R^- \notin \tau$.

The set of type models of $\mathcal{T}$ is denoted as $TM(\mathcal{T})$.

Condition 1 guarantees that a type model is also a classic model. Since $\exists R \sqsubseteq \bot$ implies $\exists R^- \sqsubseteq \bot$, we need condition 2 to guarantee that if τ is a type model of $\exists R \sqsubseteq \bot$ then it is also one of $\exists R^- \sqsubseteq \bot$. Given an DL-Lite$_{core}$ axiom ϕ, we use $TM(\phi)$ (instead of $TM(\{\phi\})$) to denote its set of type models, and models of its negation (i.e., $TM(\neg\phi)$) is defined as $\Omega_{core} \setminus TM(\phi)$. The following theorem shows that the type semantics for DL-Lite$_{core}$ TBoxes is appropriate in the sense that models of the stronger axioms are always models of the weaker ones.

Theorem 1. *Let $\mathcal{T}$ be a DL-Lite$_{core}$ TBox and ϕ an DL-Lite$_{core}$ TBox axiom. Then*

$$\mathcal{T} \models \phi \textit{ iff } TM(\mathcal{T}) \subseteq TM(\phi).$$

Proof Sketch. We prove this theorem by showing a slightly stronger result, that is, there is a strong correspondence between the type models and the DL models of a given DL-Lite$_{core}$ TBox $\mathcal{T}$. In particular, for each DL interpretation $\mathcal{I}$, a set M of types exists corresponding to $\mathcal{I}$, and $\mathcal{I}$ is a model of $\mathcal{T}$ if and only if each element of M is a type model of $\mathcal{T}$. Conversely, given a type model τ of $\mathcal{T}$, a DL model of $\mathcal{T}$ can be constructed corresponding to τ. □

For DL-Lite$_{\mathcal{R}}$, since role inclusions are allowed, a type has to include roles. A type $\tau \subseteq \mathcal{B} \cup \mathcal{R}$ for DL-Lite$_{\mathcal{R}}$ is a possibly empty set of basic concepts, atomic roles, and the inverse of atomic roles. The universal set of types for DL-Lite$_{\mathcal{R}}$ is denoted as $\Omega_{\mathcal{R}}$. If we consider basic concepts, atomic roles, inverse roles as propositional atoms, and concept inclusion $B \sqsubseteq C$ and role inclusion $R \sqsubseteq S$ as propositional formulas $\neg B \vee C$ and $\neg R \vee S$ respectively, then a DL-Lite$_{\mathcal{R}}$ TBox is a set of propositional formulas and a type for DL-Lite$_{\mathcal{R}}$ is a classic interpretation of the formulas. As for DL-Lite$_{core}$ using classic models as the semantics characterisation is inappropriate due to the non-classic inferences.

For DL-Lite$_{\mathcal{R}}$ four more non-classic inferences can be identified. First, apart from inferring $\exists R^- \sqsubseteq \bot$, $\exists R \sqsubseteq \bot$ also implies that role R and its inverse are interpreted as empty. Second, the concept inclusion $\exists R \sqsubseteq \neg\exists S$ implies the role inclusion $R \sqsubseteq \neg S$ and $R^- \sqsubseteq \neg S^-$. Third, the role inclusion $R \sqsubseteq S$ implies the concept inclusion $\exists R \sqsubseteq \exists S$, $\exists R^- \sqsubseteq \exists S^-$, and the role inclusion $R^- \sqsubseteq S^-$. Fourth. the role inclusion $R \sqsubseteq \neg S$ implies $R^- \sqsubseteq \neg S^-$. Taking into account these cases, we give below the conditions under which a type is a model of a DL-Lite$_{\mathcal{R}}$ TBox. These models will be called the type models for DL-Lite$_{\mathcal{R}}$ TBox.

Definition 2. *A type τ is a type model of a DL-Lite$_{\mathcal{R}}$ TBox $\mathcal{T}$ iff*

1. $\tau \in CM(\mathcal{T})$.
2. *if* $\exists R \sqsubseteq \bot \in cl(\mathcal{T})$ *then* $\exists R^-, R, R^- \notin \tau$.
3. *if* $\exists R \sqsubseteq \neg\exists S \in cl(\mathcal{T})$ *then* $\{R, S\} \not\subseteq \tau$ *and* $\{R^-, S^-\} \not\subseteq \tau$.
4. *for* $R, S \in \mathcal{R}$, *if* $R \sqsubseteq S \in cl(\mathcal{T})$ *then* $\exists R \in \tau$ *implies* $\exists S \in \tau$, $\exists R^- \in \tau$ *implies* $\exists S^- \in \tau$, *and* $R^- \in \tau$ *implies* $S^- \in \tau$.
5. *for* $R, S \in \mathcal{R}$, *if* $R \sqsubseteq \neg S \in cl(\mathcal{T})$ *then* $\{R^-, S^-\} \not\subseteq \tau$.

The set of type models of $\mathcal{T}$ is denoted as $TM(\mathcal{T})$.

Condition 1 guarantees that a type model is also a classic model and condition 2 to condition 5 take care of the non-classic inferences for axioms of the form $\exists R^- \sqsubseteq \bot$, $\exists R \sqsubseteq \neg\exists S$, $R \sqsubseteq S$, and $R \sqsubseteq \neg S$. Given an DL-Lite$_{\mathcal{R}}$ axiom ϕ, we use $TM(\phi)$ to denote its set of type models, models of its negation (i.e., $TM(\neg\phi)$) is defined as $\Omega_{\mathcal{R}} \setminus TM(\phi)$. The following theorem shows that type semantics for DL-Lite$_{\mathcal{R}}$ is appropriate.

Theorem 2. *Let $\mathcal{T}$ be a DL-Lite$_{\mathcal{R}}$ TBox and ϕ an DL-Lite TBox axiom. Then*

$$\mathcal{T} \models \phi \textit{ iff } TM(\mathcal{T}) \subseteq TM(\phi).$$

The theorem can be shown in a similar way as Theorem 1, while the definition of correspondence needs to be extended to involve roles.

In comparison with DL semantics, type semantics has the clear advantage of being more compact and succinct. For the TBox axiom $A \sqsubseteq B$, the three type models $\{AB\}$, $\{B\}$, and $\emptyset$ are sufficient to give the axiom a semantic characterisation. However, to give such characterisation through DL semantics, we need much more number of (DL) models as any assignment of individual to A and B such that A is a subset of B is a model of the axiom. Also when only TBox is considered the part of a DL interpretation which gives meaning to ABox assertions is in a sense redundant. A type does not hold such redundant information. Additionally, since the AGM model-based contraction and revision are defined under classic semantics and type semantics is very similar to classic semantics, it is more straight forward to adapt the AGM approach to type semantics than to do it to DL semantics.

As shown in the preliminary section, in constructing model-based contraction and revision, an intermediate set of models is first obtained then the belief set that corresponds to

Algorithm 1: $\mathcal{T}_{core}$

Input: Set of types M for DL-Lite$_{core}$
Output: DL-Lite$_{core}$ TBox $\mathcal{T}$

$\mathcal{T} := \emptyset$;
foreach $A \in \mathcal{B}$ **do**
  if *A is of the form $\exists R$ and for all $\tau \in M$, $\exists R, \exists R^- \notin \tau$* **then**
    $\mathcal{T} := \mathcal{T} \cup \{\exists R \sqsubseteq \bot\}$;
    continue;
  else if *A is atomic and for all $\tau \in M$, $A \notin \tau$* **then**
    $\mathcal{T} := \mathcal{T} \cup \{A \sqsubseteq \bot\}$;
    continue;
  foreach $B \in \mathcal{B}$ **do**
    if *for all $\tau \in M$, $A \in \tau$ implies $B \in \tau$* **then**
      $\mathcal{T} = \mathcal{T} \cup \{A \sqsubseteq B\}$;
    else if *for all $\tau \in M$, $A \in \tau$ implies $B \notin \tau$* **then**
      $\mathcal{T} := \mathcal{T} \cup \{A \sqsubseteq \neg B\}$;
return $\mathcal{T}$;

Algorithm 2: $\mathcal{T}_{\mathcal{R}}$

Input: Set of types M for DL-Lite$_{\mathcal{R}}$
Output: DL-Lite$_{\mathcal{R}}$ TBox $\mathcal{T}$

$\mathcal{T} := \emptyset$;
foreach $A \in \mathcal{B}$ **do**
  if *A is of the form $\exists R$ and for all $\tau \in M$, $\{\exists R, \exists R^-, R, R^-\} \cap \tau = \emptyset$* **then**
    $\mathcal{T} := \mathcal{T} \cup \{\exists R \sqsubseteq \bot\}$;
    continue;
  else if *A is atomic and for all $\tau \in M$, $A \notin \tau$* **then**
    $\mathcal{T} := \mathcal{T} \cup \{A \sqsubseteq \bot\}$;
    continue;
  foreach $B \in \mathcal{B}$ **do**
    if *for all $\tau \in M$, $A \in \tau$ implies $B \in \tau$* **then**
      $\mathcal{T} = \mathcal{T} \cup \{A \sqsubseteq B\}$;
    else if *A is of the form $\exists R$, B is of the form $\exists S$, and for all $\tau \in M$, $A \in \tau$ implies $B \notin \tau$, $\{R, S\} \not\subseteq \tau$ and $\{R^-, S^-\} \not\subseteq \tau$* **then**
      $\mathcal{T} := \mathcal{T} \cup \{\exists R \sqsubseteq \neg \exists S\}$;
    else if *A or B is atomic and for all $\tau \in M$, $A \in \tau$ implies $B \notin \tau$* **then**
      $\mathcal{T} := \mathcal{T} \cup \{A \sqsubseteq \neg B\}$;
foreach $R \in \mathcal{R}$ **do**
  foreach $S \in \mathcal{R}$ **do**
    if *for all $\tau \in M$, $R \in \tau$ implies $S \in \tau$, $\exists R \in \tau$ implies $\exists S \in \tau$, $\exists R^- \in \tau$ implies $\exists S^- \in \tau$, and $R^- \in \tau$ implies $S^- \in \tau$* **then**
      $\mathcal{T} := \mathcal{T} \cup \{R \sqsubseteq S\}$;
    else if *for all $\tau \in M$, $R \in \tau$ implies $S \notin \tau$ and $R^- \in \tau$ implies $S^- \notin \tau$* **then**
      $\mathcal{T} := \mathcal{T} \cup \{R \sqsubseteq \neg S\}$
return $\mathcal{T}$;

models is taken as the outcome. Under propositional logic, any set of models has a corresponding belief set. However, for contraction and revision under DLs, there may not exist a TBox or knowledge base that corresponds to the intermediate model set. Such inexpressibility results have been proved for contraction and revision under many DLs (Grau et al. 2012; Kharlamov, Zheleznyakov, and Calvanese 2013). Expressibility is also an issue with type semantics. Given a set of types M there many not exist a DL-Lite TBox whose set of type models is M.

In our view, proving inexpressibility is not the end of the story. The purpose of the intermediate model set is to give an indication of what would be the rational outcome of the contraction or revision. In case the model set has no syntactic representation under the DL considered, we should move ahead obtaining the best approximation of the model set that has an syntactic representation. Involving approximation does not undervalue the operation as the appropriateness of an operation is judged by the set of rationality postulates it satisfies.

Since our contraction and revision will be defined under type semantics, we provide the definition for best approximation of type models. For simplicity, we refer to the best approximated TBox as the corresponding one. We will show that although approximations are required, our operations satisfy all the corresponding AGM postulates.

Definition 3. *Let $\mathcal{L}$ be DL-Lite$_{core}$ or DL-Lite$_{\mathcal{R}}$. Let M be a set of type. Then $\mathcal{T}$ is the $\mathcal{L}$ TBox that corresponds to M iff $M \subseteq TM(\mathcal{T})$ and there is no $\mathcal{L}$ TBox $\mathcal{T}'$ such that $M \subseteq TM(\mathcal{T}') \subset TM(\mathcal{T})$.*

Also we provide algorithms that is $\mathcal{T}_{core}$ and $\mathcal{T}_{\mathcal{R}}$ for obtaining the best approximations for DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$. Starting with an empty TBox $\mathcal{T}$, Algorithm $\mathcal{T}_{core}$ checks if a concept inclusion holds between each pair of basic concepts, if so the concept inclusion is added to $\mathcal{T}$. A concept inclusion holds if there is no type in M that violates the inclusion. Apart from checking relations between basic concepts, Algorithm $\mathcal{T}_{\mathcal{R}}$ also checks for existence of role inclusions between each pair of roles. Since both algorithms enumerate all combinations of basic concepts or roles the returned TBox $\mathcal{T}$ is logically closed. The following proposition shows that $\mathcal{T}_{core}$ and $\mathcal{T}_{\mathcal{R}}$ are sound and complete w.r.t. Definition 3.

Proposition 1. *Let M be a set of types. Then*
1. $\mathcal{T}_{core}(M)$ is the DL-Lite$_{core}$ TBox that corresponds to M,
2. $\mathcal{T}_{\mathcal{R}}(M)$ is the DL-Lite$_{\mathcal{R}}$ TBox that corresponds to M.

In characterising the contraction operation we will work with disjunctions of TBox axioms. As such disjunction is not expressible under DL-Lite we will instead use TBox axioms that are logically stronger (under the type semantics) than the disjunction.

Definition 4. *If ϕ, ψ are DL-Lite$_{core}$ (DL-Lite$_{\mathcal{R}}$) TBox axioms then $\chi \in \mathcal{L}_{core}(\phi \vee \psi)$ (resp. $\chi \in \mathcal{L}_{\mathcal{R}}(\phi \vee \psi)$) iff χ is a DL-Lite$_{core}$ (resp. DL-Lite$_{\mathcal{R}}$) TBox axiom and $TM(\chi) \subseteq TM(\phi) \cup TM(\psi)$.*

As an example, the disjunction of DL-Lite$_{core}$ axioms $A \sqsubseteq$

B and $C \sqsubseteq D$ can be expressed as the propositional formula $\neg A \vee B \vee \neg C \vee D$. We have $\neg A$, $\neg C$, B, D, $\neg A \vee B$, $\neg C \vee D$, $\neg A \vee C$ and $\neg C \vee B$ logically stronger than $\neg A \vee B \vee \neg C \vee D$. The formulas which corresponds to the TBox axioms $A \sqsubseteq \bot$, $C \sqsubseteq \bot$, $\top \sqsubseteq B$, $\top \sqsubseteq D$, $A \sqsubseteq B$, $C \sqsubseteq D$,$A \sqsubseteq D$, and $C \sqsubseteq B$ consist of $\mathcal{L}_{core}(A \sqsubseteq B \vee C \sqsubseteq D)$. We can show that if a TBox logically implies a disjunction of axioms (non-expressible) then the TBox must also imply an axiom (expressible) that is logically stronger than the disjunction.

Lemma 1. *If ϕ, ψ are DL-Lite$_{core}$ (DL-Lite$_{\mathcal{R}}$) TBox axioms and $TM(\mathcal{T}) \subseteq TM(\phi) \cup TM(\psi)$ then there is $\chi \in \mathcal{L}_{core}(\phi \vee \psi)$ ($\chi \in \mathcal{L}_{\mathcal{R}}(\phi \vee \psi)$) such that $TM(\mathcal{T}) \subseteq TM(\chi)$.*

Contraction over DL-Lite TBoxes

In this section, we study the operation of contraction over DL-Lite TBoxes. The operation is the adaptation of AGM model-based contraction, however, instead of classic models, the approach is based on the type models defined in the previous section.

We start with adapting the AGM contraction postulates to DL-Lite. For the sake of simplicity, the postulates are used for both DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$. The TBox $\mathcal{T}$ and the axioms ϕ, ψ, and χ in the postulates are assumed to be DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$ ones accordingly. Exceptions are the postulates $(T\dot{-}5_{core})$ and $(T\dot{-}5_{\mathcal{R}})$ which are used specifically DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$ respectively.

- $(T\dot{-}1)$ $T\dot{-}\phi = cl(T\dot{-}\phi)$
- $(T\dot{-}2)$ $T\dot{-}\phi \subseteq T$
- $(T\dot{-}3)$ If $\phi \notin T$, then $T\dot{-}\phi = T$
- $(T\dot{-}4)$ If $\not\models \phi$, then $\phi \notin T\dot{-}\phi$
- $(T\dot{-}5_{core})$ If $\psi \in T$ and $\exists \chi \in \mathcal{L}_{core}(\phi \vee \psi)$ such that $\chi \in T\dot{-}\phi$ then $\psi \in T\dot{-}\phi$
- $(T\dot{-}5_{\mathcal{R}})$ If $\psi \in T$ and $\exists \chi \in \mathcal{L}_{R}(\phi \vee \psi)$ such that $\chi \in T\dot{-}\phi$ then $\psi \in T\dot{-}\phi$
- $(T\dot{-}6)$ If $\phi \equiv \psi$, then $T\dot{-}\phi = T\dot{-}\psi$
- $(T\dot{-}ct)$ If $\psi \in T\dot{-}\phi \wedge \psi$, then $\psi \in T\dot{-}\phi \wedge \psi \wedge \delta$
- $(T\dot{-}7)$ $T\dot{-}\phi \cap T\dot{-}\psi \subseteq T\dot{-}\phi \wedge \psi$
- $(T\dot{-}8)$ If $\phi \notin T\dot{-}\phi \wedge \psi$, then $T\dot{-}\phi \wedge \psi \subseteq T\dot{-}\phi$

With the exceptions of $(T\dot{-}5_{core})$, $(T\dot{-}5_{\mathcal{R}})$, and $(T\dot{-}f)$ the postulates are the direct adaptations of their AGM origins such that a belief set is now a logically closed DL-Lite TBox and the formulas are DL-Lite TBox axioms. $(T\dot{-}5_{core})$ and $(T\dot{-}5_{\mathcal{R}})$ are adaptations Disjunctive elimination. Since disjunction of axioms is not expressible in DL-Lite, their approximations are used which are obtained through the predefined operator $\mathcal{L}_{core}$ and $\mathcal{L}_{\mathcal{R}}$.

Usually, in constructing AGM contraction, a more general operation (e.g., *partial meet contraction*) is first defined that aims to satisfy the basic postulates, then a more specific operation (e.g., *transitively relational partial meet contraction*) is defined that aims for also the supplementary postulates. As in all AGM change operations, logic alone is not sufficient to determine the outcome of an operation (Gärdenfors and Rott 1995), some decision making mechanisms that are based on extralogical information are needed. For the more general operation the mechanism is assumed to be available but its details are not known whereas for the more specific operation the mechanism is made explicit by assuming certain preference information such as preorder of interpretations (as in model-based contraction and revision (Katsuno and Mendelzon 1992)), preorder of remainder sets (as in transitively relational partial meet contraction (Alchourrón and Makinson 1985)) or entrenchment of formulas (as in epistemic entrenchment contraction (Gärdenfors and Makinson 1988)).

Both the general and the specific operations are useful in removing axioms from DL TBoxes. The general one applies when we have a domain expert supervise the removing process but how the decisions are made are not known to us. The specific one applies when we want a fully automatic removing mechanism in which case some preference information has to be made explicit so as to decide on which axioms to discard and which to retain during the change. Although the AGM model-based contraction is a more specific operation in the above sense, both the general and the specific operations will be studied here.

If the model set of a TBox contains some counter models of an axiom ϕ (i.e, models of $\neg\phi$) then the TBox does not imply ϕ. Thus, to remove an axiom ϕ from a TBox $\mathcal{T}$ we can first add some counter models of ϕ to those of $\mathcal{T}$ to form an intermediate model set then obtain the corresponding TBox of the model set. So a decision has to be made on which counter models to add. For the more general contraction operation, which we call *basic model-based contraction*, a *selection function* is assumed that plays the role of decision making. Formally, γ is a selection function iff for any set of types M, $\gamma(M)$ is a non-empty subset of M unless M is empty. Essentially, the function picks from the set of counter models the "best" ones which are later added to the model set of the TBox for forming the contraction outcome.

A special case is when $\mathcal{T}$ does not imply ϕ which means the model set of $\mathcal{T}$ contains counter models of ϕ. Intuitively, if asked to remove an axiom that is not in the TBox (remember all TBoxes are assumed to be closed) then nothing has to be done, we can just return the original TBox as the outcome. In line with this intuition, a selection function is required to be *faithful* which states that if the intersection of models of $\mathcal{T}$ and those of $\neg\phi$ is not empty then the selection function returns the intersecting models and no others. In this way adding the selected counter models does not change the model set of the TBox and thus the corresponding TBox is the original one. Formally, a selection function γ is faithful with respect to a TBox $\mathcal{T}$ if for any set of types M, if $TM(\mathcal{T}) \cap M \neq \emptyset$ then $\gamma(M) = TM(\mathcal{T}) \cap M$. Faithfulness condition of AGM model-based contraction says the same thing only that it is expressed in terms of preorder of interpretations instead of selection functions.

Having faithful selection functions as the core part, basic model-based contraction for DL-Lite$_{core}$ TBox is defined as follows.

Definition 5. *A function $\dot{-}$ is a basic model-based contraction function for DL-Lite$_{core}$ TBox $\mathcal{T}$ iff for all DL-Lite$_{core}$*

TBox axioms ϕ

$$\mathcal{T}\dot{-}\phi = \mathcal{T}_{core}(TM(\mathcal{T}) \cup \gamma(TM(\neg\phi))$$

where γ *is a selection function that is faithful with respect to* $\mathcal{T}$.

We give below the representation theorem for the construction which assures that any basic model-based contraction function satisfies $(T\dot{-}1)$–$(T\dot{-}4)$, $(T\dot{-}5_{core})$, and $(T\dot{-}6)$ and any contraction function satisfying these postulates is a basic model-based contraction function.

Theorem 3. *Let* $\mathcal{T}$ *be a DL-Lite*$_{core}$ *TBox. Then* $\dot{-}$ *is a basic model-based contraction function for* $\mathcal{T}$ *iff* $\dot{-}$ *satisfies* $(T\dot{-}1)$*–*$(T\dot{-}4)$*,* $(T\dot{-}5_{core})$*, and* $(T\dot{-}6)$*.*

Proceeding to the more specific operation, which we call *model-based contraction*, the decision making has to be based on explicit preference information. For AGM model-based contraction this is in the form of preorders of classic interpretations. Here we use preorders of types. Instead of a selection function which picks the best models for us, now the top ranked (by means of the preorder) models are taken to be the best ones. With the same intuition as in the general case and as in the AGM case, the preorder has to be faithful. Formally, a preorder of types $\preceq$ is faithful with respect to a DL-Lite$_{core}$ (DL-Lite$_{\mathcal{R}}$) TBox $\mathcal{T}$ iff $min(\Omega_{core}, \preceq) = TM(\mathcal{T})$ (resp., $min(\Omega_{\mathcal{R}}, \preceq) = TM(\mathcal{T})$).

Definition 6. *A function* $\dot{-}$ *is a model-based contraction function for DL-Lite*$_{core}$ *TBox* $\mathcal{T}$ *iff for all DL-Lite*$_{core}$ *TBox axioms* ϕ

$$\mathcal{T}\dot{-}\phi = \mathcal{T}_{core}(TM(\mathcal{T}) \cup min(TM(\neg\phi), \preceq))$$

where $\preceq$ *is a faithful preorder for* $\mathcal{T}$.

By making the decision making explicit through preorders of types, the generated contraction satisfies two more postulates that is $(T\dot{-}ct)$ and $(T\dot{-}8)$. Moreover, the two are sufficient to characterise the transitive and relational nature of the preorders. Although the AGM origins of $(T\dot{-}ct)$ and $(T\dot{-}7)$ are equivalent, we can not establish their equivalence under DL-Lite. As it turns out, instead of $(T\dot{-}7)$ it is the property of $(T\dot{-}ct)$ that is needed for characterising model-based contraction over DL-Lite$_{core}$ TBoxes. The two postulates concern the contraction of conjunctions of axioms. For the axioms ϕ and ψ, their conjunction $\phi \wedge \psi$ means that both axioms hold in the TBox. Contraction of such conjunction is needed when we find out that one of them does not hold but not sure which one of them.

Theorem 4. *Let* $\mathcal{T}$ *be a DL-Lite*$_{core}$ *TBox. Then* $\dot{-}$ *is a model-based contraction function for* $\mathcal{T}$ *iff* $\dot{-}$ *satisfies* $(T\dot{-}1)$*–*$(T\dot{-}4)$*,* $(T\dot{-}5_{core})$*,* $(T\dot{-}6)$*,* $(T\dot{-}ct)$*, and* $(T\dot{-}8)$*.*

Basic model-based contraction function and model-based contraction function for DL-Lite$_{\mathcal{R}}$ TBoxes are defined as for DL-Lite$_{core}$ by replacing the operator $\mathcal{T}_{core}$ with $\mathcal{T}_{\mathcal{R}}$. Notice that we are now working with DL-Lite$_{\mathcal{R}}$ TBox axioms and the type semantics for DL-Lite$_{\mathcal{R}}$ TBoxes. The representation theorems are also identical to the DL-Lite$_{core}$ case after replacing $(T\dot{-}5_{core})$ with $(T\dot{-}5_{\mathcal{R}})$.

In addition to the representation theorems we also provide a non-deterministic algorithm for computing the contraction

Algorithm 3: CONT$_{\mathcal{R}}$

Input: DL-Lite$_R$ TBox $\mathcal{T}$ and DL-Lite$_R$ TBox axiom ϕ
Output: DL-Lite$_R$ TBox $\mathcal{T}_\phi^-$

```
1 T_φ⁻ := T;
2 Let τ ∈ TM(¬φ);
3 foreach ψ ∈ T do
4     if not SAT_R(ψ, τ) then
5         T_φ⁻ := T_φ⁻ \ {ψ};
6     else
7         continue;
8 return T_φ⁻;
```

Algorithm 4: SAT$_{\mathcal{R}}$

Input: DL-Lite$_R$ TBox axiom ϕ and type τ
Output: $True$ or $False$

```
1  if φ is of the form ∃R ⊑ ⊥ or ⊤ ⊑ ∃R then
2      return not (∃R ∈ τ or R ∈ τ or R⁻ ∈ τ);
3  else if φ is of the form ∃R ⊑ ¬∃S then
4      return not ({∃R, ∃S} ⊆ τ or {R, S} ⊆ τ or
       {R⁻, S⁻} ⊆ τ);
5  else if φ is of the form A ⊑ B and A, B ∈ B then
6      return not (A ∈ τ and B ∉ τ);
7  else if φ is of the form A ⊑ ¬B and A, B ∈ B then
8      return not (A ∈ τ and B ∈ τ);
9  else if φ is of the form R ⊑ S and R, S ∈ R then
10     return not ((R ∈ τ and S ∉ τ) or (∃R ∈ τ and
       ∃S ∉ τ) or (R⁻ ∈ τ and S⁻ ∉ τ)) or (∃R⁻ ∈ τ
       and ∃S⁻ ∉ τ);
11 else if φ is of the form R ⊑ ¬S and R, S ∈ R then
12     return not ((R ∈ τ and S ∈ τ) or (R⁻ ∈ τ and
       S⁻ ∈ τ));
```

outcome of basic model-based contraction over DL-Lite$_{\mathcal{R}}$ and DL-Lite$_{core}$ TBoxes. We first present the DL-Lite$_{\mathcal{R}}$ one.

Given a TBox axiom ϕ and a type τ, SAT$_{\mathcal{R}}$ returns $true$ if ϕ is satisfiable under τ and returns $false$ otherwise. The outcome is obtained by checking, through the if conditions in Algorithm 4, whether τ violates ϕ. The third and forth if conditions (line 5 and line 7) check for concept inclusions that incur no non-classic inferences that is their type models coincide with classic models and are specified by condition 1 of Definition 2. The rest of the if conditions check for axioms that incur non-classic inferences whose type models are specified by conditions 2-5 of Definition 2.

Given a TBox $\mathcal{T}$ and an axiom ϕ, CONT$_{\mathcal{R}}$ returns a TBox $\mathcal{T}_\phi^-$ such that $\mathcal{T}_\phi^-$ does not imply ϕ. CONT$_{\mathcal{R}}$ starts by picking any counter model τ of ϕ (line 2) then check it against each axiom in $\mathcal{T}$ (line 3). If an axiom is not satisfiable under τ (line 4) then the axiom is removed from $\mathcal{T}$ (line 5). Finally, what ever is left of $\mathcal{T}$ is returned (line 8). Proposition 2 shows that CONT$_{\mathcal{R}}$ instantiates the basic model-based contraction for DL-Lite$_{\mathcal{R}}$ TBox.

Proposition 2. *Let* $\dot{-}$ *be a contraction function for a DL-*

Lite$_{\mathcal{R}}$ TBox $\mathcal{T}$ such that $T\dot{-}\phi = CONT_{\mathcal{R}}(T,\phi)$ then $\dot{-}$ is a basic model-based contraction function for $\mathcal{T}$.

The algorithms for instantiating basic model-based contraction over DL-Lite$_{core}$ TBoxes are simplifications of Algorithm 3 and Algorithm 4 such that they discard the role inclusion part and works on DL-Lite$_{core}$ TBox axioms.

CONT$_{\mathcal{R}}$ and the corresponding operator for DL-Lite$_{core}$ run in quadratic time (if we consider the cardinality of $\mathcal{B}\cup\mathcal{R}$ linear) in the size of the TBox. In particular, obtaining a type model of $\neg\phi$ (line 2) is linear, which can be achieved by simply constructing, e.g., a type containing A but not B for $\phi = A \sqsubseteq B$, and each satisfiability check in SAT$_{\mathcal{R}}$ runs in linear time.

Revision over DL-Lite TBoxes

In this section, we study the operation of revision over DL-Lite TBoxes. In the AGM framework, revision can be constructed indirectly through contraction via the *Levi identity* (Levi 1991). Formally, let $\dot{-}$ be a contraction function for K, a revision function $*$ for K can be defined as $K * \phi = (K\dot{-}\neg\phi) + \phi$ for all formulas ϕ. Since negations of DLs axioms are not always expressible, defining DL revision from a DL contraction is no simple matter. Studying such definability results is not in the scope of this paper. As in (Katsuno and Mendelzon 1992), we take the direct approach in defining revision.

We first clarify a fundamental difference between AGM revision and revision over DL TBoxes (DL revision). AGM revision aims to incorporate a new formula to the belief set while resolving any inconsistency caused. The AGM revision postulates are formulated to capture the rationale behind the inconsistency resolving process. DL revision goes beyond inconsistency resolving. In addition to consistency, meaningful DL TBoxes have to be coherent, thus DL revision also has to resolve any incoherence caused in incorporating new axioms. For this reason, the postulates for DL revision have to be reformulated from the AGM ones to capture the rationale behind not only inconsistency but also incoherence resolving.

In fact we can concentrate on coherence resolving when ABox is not considered. By its definition, a coherent TBox must be consistent. Inconsistency resolving is thus part of incoherence resolving.

For the sake of simplicity, we extend the notion of coherence to single TBox axioms and sets of types. An axiom ϕ (a set of types M) is coherent iff $TM(\phi) \not\subseteq TM(B \sqsubseteq \bot)$ (resp. $M \not\subseteq TM(B \sqsubseteq \bot)$) for all basic concepts B.

Adapting from the AGM revision postulates by replacing conditions on consistency with coherence, we obtain the following postulates for revision over DL-Lite TBoxes.

$(T * 1)$ $T * \phi = cl(T * \phi)$

$(T * 2)$ $\phi \in T * \phi$

$(T * 3)$ If ϕ is coherent then $T * \phi \subseteq T + \phi$

$(T * 4)$ If $T \cup \{\phi\}$ is coherent then $T + \phi \subseteq T * \phi$

$(T * 5)$ If ϕ is coherent then $T * \phi$ is coherent

$(T * 6)$ If $\phi \equiv \psi$ then $K * \phi = K * \psi$

$(T * f)$ If ϕ is incoherent then $T * \phi = \{\top \sqsubseteq \bot\}$

The failure postulate $(T * f)$ is dedicated to the limiting case when the revising axiom is incoherent in which case we return the inconsistent TBox. Its AGM origin, which states if the revising formula is inconsistent then we return the inconsistent belief set, is deducible from $(K * 2)$ thus is not postulated explicitly. As for contraction, the postulates are overloaded for both DL-Lite$_{core}$ and DL-Lite$_{\mathcal{R}}$. For reasons that will be explained later supplementary postulates are not considered here.

If the model set of a TBox $\mathcal{T}$ is the subset of that of an axiom ϕ then $\mathcal{T}$ implies ϕ. Thus to incorporate an axiom ϕ to a TBox $\mathcal{T}$, we can pick some models of ϕ to form an intermediate model set then obtain its corresponding TBox. So a decision has to be made on which models of ϕ to pick. As for contraction, a selection function is assumed.

Since we are in the realm of incoherent handling, the definition of selection function has to be modified accordingly. Previously, for contraction, a selection function returns empty set if the input is empty which represents the limiting case when the contracting axiom is a tautology (thus its negation is inconsistent and has an empty model set). Now the limiting case is when the revising axiom is incoherent. As along as $(T * 2)$ is concerned, there is no way to return a coherent TBox in this case and as required by $(T * f)$ we return the inconsistent TBox. Formally, a function γ is a selection function if $\gamma(M)$ is a non-empty subset of M unless M is incoherent.

The faithfulness condition also has to be modified. A selection function γ is *faithful* with respect to a TBox $\mathcal{T}$ if it satisfies

1. if M is coherent then $TM(\mathcal{T}) \cap M \subseteq \gamma(M)$, and
2. if $TM(\mathcal{T}) \cap M$ is coherent then $\gamma(M) = TM(\mathcal{T}) \cap M$.

In revising $\mathcal{T}$ by ϕ, condition 1 deals with the case when models of $\mathcal{T}$ overlaps with those of ϕ which means $\mathcal{T} \cup \{\phi\}$ is consistent. In line with the principle of minimal change, in this case, the selection function has to pick all the overlapping models to preserve as much as possible the original TBox axioms. Condition 2 deals with the case that not only the overlapping exists but also it is coherent. Since there is no incoherence to resolve, the revision boils down to expansion (i.e., $cl(\mathcal{T} \cup \{\phi\})$). The selection function therefore picks all the overlapping models and no others.

Central to the revision, the selection function has to guarantee the type models picked are coherent, thus the following condition. A selection function γ is *coherent preserving* if $\gamma(M)$ is coherent whenever M is so.

Determined by a coherent preserving and faithful selection function, a *basic model-based revision* function for DL-Lite$_{core}$ TBoxes is defined as follows. And as shown in Theorem 5, such functions can be characterised by $(T * 1)$–$(T * 6)$ and $(T * f)$. The definition and characterisation for DL-Lite$_{\mathcal{R}}$ TBoxes are identical to the DL-Lite$_{core}$ case after replacing $\mathcal{T}_{core}$ with $\mathcal{T}_{\mathcal{R}}$.

Definition 7. *A function $*$ is a basic model-based revision function for DL-Lite$_{core}$ TBox $\mathcal{T}$ iff for all DL-Lite$_{core}$ TBox axioms ϕ*

$$\mathcal{T} * \phi = \mathcal{T}_{core}(\gamma(TM(\phi)))$$

where γ is a selection function that is coherent preserving and faithful with respect to $\mathcal{T}$.

Theorem 5. *Let $\mathcal{T}$ be a DL-Lite$_{core}$ TBox. Then $*$ is a basic model-based revision function for $\mathcal{T}$ iff $*$ satisfies $(T*1)$–$(T*6)$ and $(T*f)$.*

Basic model-based contraction for DL-Lite TBox can be specialised by requiring the transitivity and relationality of the selection function through a preorder of types. The top ranked types in the preorder are then used for forming the contraction outcome. The specialised contraction is better behaved than the basic one in the sense that it satisfies two more postulates (i.e., $(T\dot{-}ct)$ and $(T\dot{-}8)$) that are intuitively plausible. However, the task of specialising basic model-based revision is challenging.

A preorder over types is no longer appropriate, as the set of top ranked types may not be coherent. What we need is a preorder over coherent sets of types. In other words each element in the rank is a coherent set of types. The AGM postulates $(K*7)$ and $(K*8)$ which characterise the transitivity and relationally of a preorder over single models is incompatible for preorders over sets of models. A long term goal is to identify the supplementary postulates, if there exists any, for characterising revisions determined by preorders of sets of models.

Related Work

In defining DL contraction and revision, our work and those of (Qi and Du 2009; Wang, Wang, and Topor 2010) consider changes over logically closed TBoxes and knowledge bases. Qi and Du (Qi and Du 2009) approached DL revision model-theoretically by adapting Dalal's revision operator (Dalal 1988). The work is independent of a specific DL. Postulates for their operators are however not adapted appropriately for handling incoherence. Wang et al (Wang, Wang, and Topor 2010) adapted Satoh's revision operator (Satoh 1988) for revision over DL-Lite knowledge bases. Similar to ours, they also use an alternative semantics called feature. However, unlike type semantics that resembles with the propositional case, a feature in their characterisation is of exponential size (w.r.t. the TBox). The adapted operators cannot guarantee coherence in general.

Some works such as those of (Qi et al. 2008; Ribeiro and Wassermann 2009) consider changes over TBoxes and knowledge bases that are not necessarily closed. Both works are independent of a specific DL. Qi et al (Qi et al. 2008) adapted and implemented *kernel revision* (Hansson 1994). Ribeiro and Wassermann (Ribeiro and Wassermann 2009) adapted *partial meet revision* and *semi-revision* (Hansson 1997). In these works, only the axioms explicitly present in the TBox or knowledge base are considered. The implicit axioms which logically follow from the explicit ones but are not present are discarded during the operation. As we work with closed TBoxes, our operations take into account both the explicit and implicit axioms. As emphasized in (Grau et al. 2012), an optimal change operation in DL should maximally preserve the implicit axioms.

The work of (Grau et al. 2012) take a combined approach that contracts and revises at the same time. A constraint which states the set of axioms to be incorporated and those to be eliminated is first specified. Then a combined operation maps a knowledge base to another that satisfies the constraint. The operation reduces to a revision and contraction after making empty the eliminating set and the incorporating set respectively. Under DL-Lite and when considering only TBoxes, the contraction is a restricted form of our basic model-based contraction. There is no explicit mention that the revision considers incoherence resolving.

The work of (Kharlamov, Zheleznyakov, and Calvanese 2013) which summarises several previous works of the authors studies model-based revision and update over DL-Lite knowledge bases such that only ABoxes are allowed to change. Different from ours, the focus is on the expressibility of the revision and update outcomes.

Going beyond DLs, the work of (Ribeiro et al. 2013) which summarises several previous works of the authors identifies the properties of a particular logic under which a *partial meet contraction* satisfies Recovery or Relevance. The investigation shows that the majority of DLs including DL-Lite are *relevance-compliant* but not *recovery-compliant* meaning that contractions can be defined under these logics that satisfy Relevance but not Recovery. Our model-based contractions do not satisfy relevance instead they satisfy the other alternative of Recovery that is Disjunctive elimination. As stated in (Ribeiro et al. 2013), they only considered the basic postulates and defining a contraction that also satisfies the supplementary ones in their general setting is proven difficult. Our representation theorems show that extending to supplementary postulates is possible for DL-Lite given that we stick to Disjunctive elimination.

Conclusion

In this paper, we proposed type semantics for the inferences of DL-Lite TBoxes which outperforms DL semantics in the compactness of characterising DL-Lite TBoxes and suitability of adapting AGM style change operation. Assuming type semantics, we defined and characterised model-based contraction and revision over DL-Lite TBoxes.

The contraction is defined as in the AGM case. The representation theorems suggest that, with proper adaptation, the results for AGM contraction which essentially assumes propositional logic are generalizable to DL-Lite. Defining revision is trickier than defining contraction such that we only tackled the more general operation. DL revision differs intrinsically from AGM revision such that the former not only deals with inconsistency but also incoherence. Thus AGM revision postulates and constructions have to be adapted to coherence centred. The remaining problems are the proper construction and characterisation of the more specific operation.

In addition to the open problems, we also plan to study contraction and revision over DLs that allow existential or universal quantifiers. Since concepts of infinite length can be formed in these DLs through unbound nesting of quantifies, their semantic characterisation through type semantics may not be possible. We need some other techniques that are tailored to these DLs.

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