Response to the comments made by Reviewers to
the manuscript “Approaches to Measuring
Inconsistency for Stratified Knowledge Bases” by
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We wish to thank the two reviewers for their valuable comments. We have addressed all their comments and revised the manuscript.
Note that all of the revisions are highlighted in Blue in the revised manuscript.

Detailed Responses to Reviewers Comments

The comments by the reviewers are in italic and our responses are as plain text.

1 Comments by Reviewer 1

- My main concern with the paper is that whilst a range of options are investigated for measuring inconsistency in prioritized knowledge bases, each of which is significantly influenced by the measures proposed in the literature for flat knowledge bases, there is a lack of a unified framework for comparing the different options. The results only show some differences between them in terms of some logical properties and computational properties. For instance, there is a lack of results that would enable the reader to determine that one kind of measure is better for some situations than others.

— We have reformulated a set of desirable properties for measuring inconsistency in the setting of stratified KBs, such as Consistency, Monotony, Domi-nance, Safe Formula Independence, and Inferior Stratun. This is essentially a general framework for characterizing different inconsistency measures for strat-ified KBs. We have also compared the computational complexities of various inconsistency measures. In addition, for each inconsistency measure, we have analyzed or explained its goal and focus. To highlight these comparative results,
– We added the following sentence to the first paragraph of Section 3 (in page 9) for further explaining the first approach:

Any inconsistency measure defined by this approach not only describes all the inconsistencies in a stratified knowledge base, but also explicitly provides an assessment for the inconsistency arising in each section of the base.

– We added the following sentences to the first paragraph in page 49:

Such an inconsistency measure is more appropriate for looking inside an inconsistent stratified knowledge base stratum by stratum. Generally, the naive MSIM is more appropriate for describing how inconsistent each cut of a knowledge base is, whilst the stratum-centrical MSIM is more suitable for uncovering how preferred the strata involved in inconsistency are at each stratum.

We hope these can make sense.

• A subsidiary concern is that no consideration has been given to the nature of prioritized knowledge bases as explored in particular by Gerd Brewka (Gerhard Brewka: Preferred Subtheories: An Extended Logical Framework for Default Reasoning. IJCAI 1989: 1043-1048), and also by Salem Benferhat et al. I think some comparison between the measures based on MI and Brewka’s preferred subtheories would provide some useful insights into those measures.

— At first, we cannot agree that no consideration has been given to the nature of prioritized knowledge bases. Both the vectorial structure and the Inferior stratum property are proposed to ensure that the nature of prioritized knowledge bases are considered adequately in defining an inconsistency measure. Second, Brewka’s preferred subtheories and our approaches are technically different. Informally speaking, preferred subtheories provides a way to obtain ”good” information from inconsistent knowledge base, whilst our approaches focus on describing how ”bad”(inconsistent) the knowledge base is. They have different goals and focus on different parts. Preferred subtheories may not be considered relevant to inconsistency characterization explicitly. To illustrate this, consider $K_1 = (\{a\}, \{b, \neg a\}, \{c\})$ and $K_2 = (\{a\}, \{b\}, \{\neg a, c\})$. Intuitively, $K_1$ is more inconsistent than $K_2$, but they have the same preferred subtheory ($\{a\}, \{b\}, \{c\}$).

• Section 2. ”propositional symbols” and ”propositional variables” are the same thing. So just use one term, not both.

— We have replaced propositional symbols with propositional variables.

• Section 2. Using bold font to differentiate some symbols is not helpful as the reader may fail to notice the difference when used which makes it confusing.

— We have replaced bold letters $c, d, \cdots$ with $\vec{c}, \vec{d}, \cdots$, respectively.
• Section 2. "reflective" should be "reflexive"

— We have corrected it.

• Page 5, Line 46. $C_K$ is a tuple, but it is used incorrectly here as a set.

— As stated explicitly in Section 2, we use $C_K$ to denote the set of formulas of a stratified knowledge base $K$ in this paper. $C_K$ is not used to denote a tuple anywhere in this paper.

• Page 5, line 53. "in some sense" but surely they are the same thing, and so this phrase is misleading.

— We have deleted it.

• Page 6, first sentence. This sentence is confusing as a definition. I think it could be rewritten.

— We have revised it as follows:

Given a stratified knowledge base $K = (S_1, S_2, \ldots, S_n)$ and an integer $i$ ($1 \leq i \leq n$), we use $K \cup_i \{\alpha\}$ to denote the stratified knowledge base obtained by adding $\alpha$ to the $i$-th stratum of $K$, i.e., $K \cup_i \{\alpha\} = (S_1, S_2, \ldots, S_i \cup \{\alpha\}, \ldots, S_n)$.

• Page 6, line 16. "for convenient" should be "for convenience"

— We have corrected it.

• Page 6, line 25. What is "L" in "LMI"?

— To clarify this, we add the following sentence to the last paragraph of Section 2.1 in page 6:

Here $L$ in LMI means lexicographical ordering, which is used to define $lc(K)$.

• Page 8, line 27. Why say "in particular" here?

— We have replaced “in particular” with “furthermore”.

• Page 9, line 28. The difference between the two types of measure is not clear from the explanation in this sentence. It would be useful to the reader to have a better sentence.
Any inconsistency measure defined by this approach not only describes all the inconsistencies in a stratified knowledge base, but also explicitly provides an assessment for the inconsistency arising in each section of the base.

We hope this explanation is clear.

• Page 9, line 34. "particularly" should be "particular"

—We have corrected this.

• Definition 3.1. This definition seems too weak to be meaningful.

— This definition describes a framework for characterizing an inconsistency measure. To clarify this, we have revised Definition 3.1 as follows:

Then it is natural to define such a framework for measuring inconsistency for stratified knowledge bases in the following way.

**Definition 1.1 (Multi-section inconsistency measure)** Let $K = (S_1, S_2, \cdots, S_n)$ be a stratified knowledge base and $\text{Inc}_m$ an inconsistency measure for stratified knowledge bases. Then $\text{Inc}_m(K)$ is called a multi-section inconsistency measure (MSIM for short) for $K$ if

$$\text{Inc}_m(K) = (c_1, c_2, \cdots, c_n)$$

such that

$$\text{Inc}_m(K_{1\to i}) = (c_1, c_2, \cdots, c_i)$$

for all $1 \leq i \leq n$.

• Definition 3.5. This definition together with the subsidiary definitions on which it is based are quite complicated. Is it really not possible to define this notion in simpler way?

— We have tried to make Definition 3.5 and related notions less complicated. In detail, we have rephrased several paragraphs after Example 3.3 as follows:

In the above example, $M_2 = \{b, \neg b\}$ is the only minimal inconsistent subset relevant to $S_3$. Moreover, $S_2$ is the most preferred stratum involved in $M_2$. Then we can use the level number 2 to characterize the degree of inconsistency partially caused by $S_3$. In general, we can formalize this notion as follows.

Given a stratified KB $K = (S_1, S_2, \cdots, S_n)$ and a fixed level $i$ with $\text{Mis}(S_i) \neq \emptyset$ ($1 \leq i \leq n$), we say that the $k$-th stratum $S_k$ ($k \leq i$) is involved in inconsistency due to $S_i$ if $S_k \cap M \neq \emptyset$ for some $M \in \text{Mis}(S_i)$. We use $\kappa(S_i)$ to denote
the minimum level of strata involved in minimal inconsistent subsets due to $S_i$. For convenience, we set $\kappa(S_i) = i + 1$ if $\text{Mis}(S_i) = \emptyset$. Formally,

$$\kappa(S_i) = \begin{cases} 
\min \{1 \leq k \leq i | \exists M \in \text{Mis}(S_i) \text{ s.t. } S_k \cap M \neq \emptyset\}, & \text{if } \text{Mis}(S_i) \neq \emptyset; \\
i + 1, & \text{otherwise}.
\end{cases}$$

Recall the three knowledge bases illustrated in Figure 2, then $\kappa(S_3) = 3$ for $K_1$, $\kappa(S_3) = 2$ for $K_2$, and $\kappa(S_3) = 1$ for $K_3$. It can make a distinct among the three knowledge bases.

We can use the number $\kappa(S_i)$ to characterize the inconsistency due to $S_i$. However, for technical reasons, we introduce the following multi-section inconsistency measure.

- **Example 3.3.** The intuition of definition 3.5 seems strange when looking at these examples. For instance, for $K_1$ which is based on only two propositional variables, and contains only two minimal inconsistent subsets, has a value of 3 for the third stratum. What does 3 mean here?

  — Example 3.3 is correct. Here the value of 3 for the third stratum means that the first stratum is involved in inconsistency partially caused by the third stratum.

- **Example 4.5, line 31.** Delete ”in”

  — We have corrected this.

- **Example 4.6.** Something seems incorrect here. For instance, for $K_1$, which has tuple $(0,0,2)$ in Example 3.2, has the evaluation $w_1 + w_2 + 2w_3$. I think it should be $2w_3$.

  — Example 4.6 is correct. Example 3.2 and Example 4.6 use different measures. However, Example 4.6 is based on the results of Example 4.2, in which $K_1$ has measure $(1, 1, 2)$.

- **Proposition 5.1 and 5.2.** I do not understand why the layout for these propositions is different. The first uses the name as given in the definitions of the axioms, whereas the second gives the specific instance of the axioms. If they need to be like this, then some explanation would be helpful, otherwise a standard format would be better.

  — We have rephrased related Propositions. Now they are uniform in the layout.

- **Proof of monotony in Proposition 5.2, line 40.** I don’t see how $\kappa(S_j) \geq \kappa(T_j)$ is obtained here. Adding a formula level $i$ may create a new minimal inconsistent set that involves a formula in a more preferred strata, and therefore in that case how $\kappa(S_j) \leq \kappa(T_j)$.
— The proof is correct. It can be derived from definition of $\kappa$ directly.

- Section 7. "system-to-be" should be "system".

— “System-to-be” is the usual term in requirements engineering. It is referred to as the system to be developed.

- References. Full book titles required for refs 4, 21, and 25. Also, it would be good to reference other papers published in IJAR on the topic of measures of inconsistency.

— We have corrected this, and added the following two new references:


2 Comments by Reviewer 2

- To some extent, the reviewer would have prefered a more compact paper (but this cannot be understood as a criticism towards the authors who did much to make everything clear as spring water; the price being the length of the paper).

— We have tried to make the paper compact. We have simplified Propositions 5.2,5.4 and 5.6 in the layout.