1. Prove informally and by mathematical induction that every finite language is regular.

2. Write regular expressions for the following languages:
   (a) The set of strings over \{a, b, c\} with at least one a and at least one b.
   (b) The set of strings of 0s and 1s with at most one occurrence of 11.
   (c) The set of strings of 0s and 1s that does not contain 00 or 101.
   (d) The set of strings of 0s and 1s with an even number of 0s that end in 101.

3. Write a regular expression that captures the different forms of Australian phone numbers, e.g. +61 7 3875 5047, (07) 3875 5047, 07 3875 5047, 3875 5047.

4. State which of the following statements about regular expressions is true and which is false, and justify your answers:
   (a) \( R(S + T) = RS + RT \)
   (b) \((R + S)^* = (R^*S^*)^*\)
   (c) \((R + S)^* = R^* + S^*\)
   (d) \((RS + R)^*R = R(SR + R)^*\)
   (e) \((RS + R)^*RS = (R^+S)^*\)
   (f) \((R + S)^*S = (R^*S)^*\)
   (g) \(S(RS + S)^*R = R^+S(R^+S)^*\)

   To justify a claim that a statement is true, an (informal) proof is required; to justify a claim that a statement is false, a single counterexample is sufficient. (IALC, Exercise 3.4.2)

5. Show two different, simpler, equivalent expressions for
   \((0 + 1)^*1(0 + 1) + (0 + 1)^*1(0 + 1)(0 + 1)\).

   (IALC, Exercise 3.4.3)

6. Construct a DFA that recognises the language in \(\{0, 1\}^*\) of (binary) numbers evenly divisible by three.

7. Construct a regular expression that defines the language in the previous question.

8. For \(N \geq 1\), let
   \[ L_N = \{ s \in \{0, 1\}^* \mid |s| \geq N \text{ and the } N\text{th symbol from the right is } 1 \} \]

   Construct an NFA that recognises \(L_3\).
9. Construct a DFA that recognises the language in the previous question.

10. (a) Given a DFA that recognises a language \( L \), construct a DFA that recognises the complement \( L' \) of \( L \).
    (b) Given DFAs that recognise languages \( L_1 \) and \( L_2 \), construct a DFA that recognises the symmetric difference \( L_1 \Delta L_2 \) of \( L_1 \) and \( L_2 \).

11. If \( L \) is a language and \( a \) a symbol, then define \( L/a \) to be the set of strings \( w \) such that \( wa \in L \). For example, if \( L \) is the language of the regular expression \((01)^*\), then \( L/1 \) has the regular expression \((01)^*0\) and \( L/0 \) is empty. Prove that if \( L \) is regular, then so is \( L/a \). (Hint: Use a construction based on a DFA for \( L \).)

12. Transform the following regular expressions to equivalent NFAs:
    (a) \( \emptyset^* \)
    (b) \( 01^*0 \)
    (c) \( 00(01 + 1)^*1 \)
    (d) \( (((00)^*11) + 01)^* \)

13. Transform each of the NFAs in IALC, Exercises 2.3.1 to 2.3.3, to equivalent DFAs (and informally describe the language recognised in each case).

14. Transform each of the NFAs in Sipser, Exercise 1.16, to equivalent DFAs (and informally describe the language recognised in each case).

15. Construct a regular expression for the set of strings in \( \{0, 1\}^* \) that, when interpreted in \( \text{reverse} \) as a binary number, are divisible by 3.

16. Construct a regular expression for the set of strings in \( \{0, 1\}^* \) that, when interpreted as a binary number, are divisible by 5.

17. Construct a 4-state DFA for the set of strings in \( \{0, 1\}^* \) that contain an even number of 0s and an even number of 1s. Use the state-elimination construction to construct an equivalent regular expression.

18. Construct equivalent regular expressions for the DFAs in IALC, Exercises 3.2.1 to 3.2.3.

19. Construct equivalent regular expressions for the DFAs in Sipser, Exercise 1.21.

20. Use the pumping lemma for regular languages to prove that none of the following languages is regular:
    (a) \( \{ 0^n1^n \mid n \geq 1 \} \)
    (b) \( \{ 0^n1^m \mid n \leq m \} \)
    (c) \( \{ ww \mid w \in \{a, b\}^* \} \)
    (d) \( \{ wtw \mid w, t \in \{a, b\}^* \} \)
    (e) \( \{ 0^n \mid n \text{ is a power of 2} \} \)
21. Let $\Sigma = \{0, 1, +, =\}$ and 

$$A = \{x + y = z \mid x, y, z \text{ are binary integers and } z \text{ is the sum of } x \text{ and } y\}.$$ 

Prove that $A$ is not regular.

22. Describe what happens when you attempt to use the pumping lemma to show that some finite (and hence regular) language is not regular.

23. Minimise each of the following two DFAs:

(a) 

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(b) (IALC, Exercises 4.4.1 and 4.4.2)

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24. Describe decision algorithms to answer each of the questions in IALC, Exercises 4.3.3 to 4.3.5.
