3515ICT Theory of Computation

Take-home test 3 (2007)

(40 marks)

1. Give a detailed description of a possible fixed-alphabet encoding $\langle M \rangle$ of a Turing machine M. The encoding must be able to represent any M, no matter how many states it has, how large its tape alphabet is, or how many transitions it has.

   (4 marks)

2. Give an implementation-level description of a (deterministic) universal Turing machine U, *i.e.*, a Turing machine that takes an encoding $\langle M, w \rangle$ of a Turing machine M and an input string w as its input, and simulates the behaviour of M on w. The machine U may use multiple tapes.

   For simplicity in this question, you may assume the tape alphabet of each TM M is a subset of the tape alphabet of U. Carefully state any other assumptions you make.

   Bonus marks will be awarded for implementing and testing your universal Turing machine on one of the Turing machine simulators mentioned under Resources.

   (16 marks)

3. Let k be a positive integer, let $\alpha_1,\ldots,\alpha_k$ be nonempty strings in Σ^+, and let $c_1,\ldots,c_k$ be symbols *not* in Σ.

   Let L_A be the context-free language generated by the following grammar.

   $$A \to \alpha_1 c_1 \mid \alpha_2 c_2 \mid \cdots \mid \alpha_k c_k \mid \alpha_1 A c_1 \mid \alpha_2 A c_2 \mid \cdots \mid \alpha_k A c_k$$

   (Sentences in L_A are effectively nested pairs of parentheses of different types, where each left "parenthesis" may consist of several symbols.)

   Prove that the complement $\overline{L_A}$ of L_A is also a context-free language by giving a pushdown automaton that recognises $\overline{L_A}$.

   (4 marks)

4. For each of the following instances of the Post Correspondence Problem either give a solution or show that no solution exists.

   $$\left\{ \left[\frac{100}{10}\right], \left[\frac{101}{01}\right], \left[\frac{110}{1010}\right] \right\}$$

   $$\left\{ \left[\frac{1}{10}\right], \left[\frac{01}{101}\right], \left[\frac{0}{101}\right], \left[\frac{001}{0}\right] \right\}$$

   (4 marks)

5. Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

   (4 marks)

6. Prove that each of the following two languages is undecidable.

   (a) $A_{1} = \{ \langle M \rangle \mid M$ is a TM that when started with a blank tape eventually writes a 1 somewhere on its tape $\}$

   (b) $A_{1011} = \{ \langle M \rangle \mid M$ is a TM and $1011 \in L(M) \}$.

   (4 marks)

7. By Rice's Theorem, the following properties of Turing machines are both undecidable. But are they Turing-recognisable or not?

   (a) Does $L(M)$ contain at least two strings?

   (b) Is $L(M)$ finite?

   (4 marks)

Submission

Prepare your solution on A4 pages, stapled in top-left corner, without other bindings. Include appropriate identification. Write neatly, handwriting is OK, black or blue ink is preferred.

Clear, simple, neatly presented solutions will receive more marks than unclear, complicated, untidy solutions. Show all working.

Put your solution in my (Rodney's) letter box in level 0 of N44.

Due date

6pm on Thursday 4 October 2007.

Late penalties

10% per weekday until Thursday 11 October 2007.