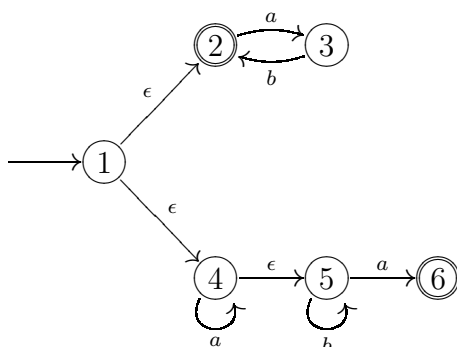


3515ICT Theory of Computation

Take-home test 2

Each question is worth 4 marks.

- Convert the following NFA to a DFA, using the subset construction algorithm.



- Construct the minimal DFA for the following DFA using the state equivalence algorithm.

	a	b
→ 1	2	3
2	4	5
3	6	7
*4	4	5
5	6	7
*6	4	5
*7	6	7

- Use the pumping lemma for regular languages to prove that following language is not regular:

$$L_1 = \{ a^n b^{2n} \mid n \geq 0 \}$$

- Give context-free grammars that generate the following languages.

(a) $\{ a^m b^n c^n \mid m, n \geq 0 \}$

(b) $\{ a^i b^j c^k \mid i = j + k \}$

- Consider the following grammar G over the alphabet $\{a, b, c\}$.

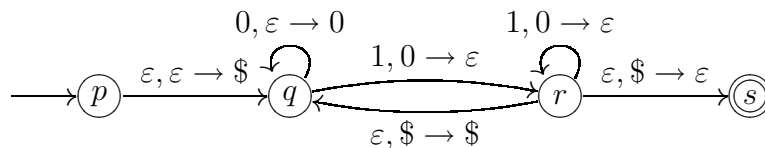
$$S \rightarrow c \mid aS \mid aSbS$$

- Informally describe $L(G)$.
- Prove that G is ambiguous.
- Give an unambiguous grammar for $L(G)$.

6. Give pushdown automata that recognise the following languages.
- (a) $\{ w \in \{a, b\}^* \mid w = w^R \}$ (Note that $|w|$ may be even or odd.)
 - (b) $\{ a^i b^j c^k \mid i < j \text{ or } j < k \}$
7. (a) Give an example of a language that is not deterministic, *i.e.*, that cannot be recognised by any deterministic PDA.
- (b) Give a deterministic PDA that recognises the language of balanced parenthesis strings.
8. Use the pumping lemma for context-free languages to prove the following language is not context-free:

$$L_2 = \{ a^{n^2} \mid n \geq 1 \}$$

9. (a) Suppose that L is a context-free language and that R is a regular language. Prove that $L \setminus R$ is a context-free language.
- (b) Give a PDA that recognises the set of palindromes in $\{a, b\}^*$ such that the number of a 's is a multiple of 3. (Note that this set includes both odd- and even-length palindromes.) There is a standard construction that can be used to solve this problem.
10. Consider the following PDA:



Use the general transformation from a pushdown automaton to a context-free grammar to construct a context-free grammar that generates the language this automaton recognises.

Submission

Prepare your solution on A4 pages, stapled in top-left corner, without other bindings. Include appropriate identification. Write neatly in black or blue ink, handwriting is OK.

Clear, simple, neatly presented solutions will receive more marks than unclear, complicated, untidy solutions. Show all working.

Due date

4pm on Monday 10 September 2007. Give your solution to me at the lecture. Alternatively, put it in my mailbox on level 0 of N44 or email it to me if you can't attend the class.

Late penalties

10% per day until Friday 14 September 2007.

Rodney Topor