

# 3515ICT Theory of Computation

## Take-home test 1

Every question is worth 1 mark except where indicated.

1. Prove that 133 divides  $11^{n+1} + 12^{2n-1}$  for all  $n \geq 1$ ,
2. Prove that the set of all full binary trees whose leaves are  $a$  or  $b$  is countably infinite. (Describe a 1-1 onto mapping from the set of natural numbers to the set of trees; do not use induction.)
3. Construct regular expressions for each of the following subsets of  $\{0, 1\}^*$ :
  - (a) The set of strings that do **not** end with 01.
  - (b) The set of strings that contain an even number of 0s and end with 1.
4. Construct DFAs over the alphabet  $\{0, 1\}$  that recognise each of the following languages:
  - (a) The set of strings containing 000.
  - (b) The set of strings that contain at least one 0 and whose number of 1s is evenly divisible by 3.
5. Construct **simple** NFAs over the alphabet  $\{0, 1\}$  that recognise each of the following languages:
  - (a) The set of strings containing 000.
  - (b) The set of strings consisting of either 01 repeated one or more times or 010 repeated one or more times.
6. Transform the following regular expression to an equivalent NFA:

$$(ab)^*c + a(bc)^*$$

7. Transform the following NFA to an equivalent DFA, using the subset construction algorithm.

	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
$q$	$\{r, s\}$	$\{t\}$
$r$	$\{p, r\}$	$\{t\}$
$*s$	$\emptyset$	$\emptyset$
$*t$	$\emptyset$	$\emptyset$

(2 marks)

8. Transform the following DFA to an equivalent regular expression, using the state elimination algorithm.

	0	1
$\rightarrow$ *p	s	p
q	p	s
r	r	q
s	q	r

(2 marks)

### Submission

Prepare your solution on A4 pages, stapled in top-left corner, without other bindings. Include appropriate identification. Write neatly in black or blue ink; handwriting is OK.

Clear, simple, neatly presented solutions will receive more marks than unclear, complicated, untidy solutions.

### Due date

4pm on Monday 13 August 2007. Give your solution to me at the lecture. Alternatively, put it in my mailbox on level 0 of N44 or email it to me if you can't attend the class.

Rodney Topor