1. State whether or not each of the following languages is regular.
(a) The set of strings in \( \{0, 1\}^* \) with an even number of 0s.
(b) The set of strings in \( \{0, 1\}^* \) with a prime number of 0s.
(c) The set of strings in \( \{0, 1\}^* \) with twice as many 0s as 1s.
(d) The set of strings in \( \{0, 1\}^* \) with at least three 0s and at most two 1s.

State whether or not each of the following languages is regular.

(e) \( L_1 \cap L_2 \), where \( L_1 \) and \( L_2 \) are regular languages.
(f) \( L^R = \{ w^R \mid w \in L \} \), where \( L \) is a regular language, and \((a_1 \ldots a_n)^R = a_n \ldots a_1\).

State whether or not each of the following problems is decidable. If it is, briefly explain why. If not, briefly explain why not.

(g) Given \( M_1 \) and \( M_2 \) are finite automata, is there a string that belongs to both \( L(M_1) \) and \( L(M_2) \)?

(h) Given \( M_1 \) and \( M_2 \) are finite automata, is \( L(M_1) \) a subset of \( L(M_2) \)?

(10 marks)

2. (a) Give a regular expression that defines the set of strings in \( \{0, 1\}^* \) that do not contain 00. Example strings include \( \epsilon \), 0, 01 and 11010.

(b) Transform the following deterministic finite automaton \( M_1 \) into an equivalent regular expression \( E \) (i.e., so that \( E \) defines \( L(M_1) \)).

\[
\begin{array}{c|cc}
   & a & b \\
\hline
\rightarrow p & q & q \\
\star q & q & r \\
\star r & q & p \\
\end{array}
\]
(Note there are two accepting states.)

(10 marks)
3. Consider the following nondeterministic finite automaton $M_2$.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow p$</td>
<td>${q, r}$</td>
<td>${s}$</td>
</tr>
<tr>
<td>$*q$</td>
<td>$\emptyset$</td>
<td>${q}$</td>
</tr>
<tr>
<td>$r$</td>
<td>${r}$</td>
<td>${s}$</td>
</tr>
<tr>
<td>$*s$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

(Note there are two accepting states.)

(a) Show the value of $\delta^*(p, ab)$.

(b) Transform $M_2$ into an equivalent deterministic finite automaton.

(10 marks)

4. Let $\Sigma = \{0, 1, +, =\}$ be an alphabet and

$$L = \{ a + b = c \mid a, b, c \text{ are binary integers, and } c \text{ is the sum of } a \text{ and } b \}$$

a language over $\Sigma$. For example, $L$ contains the strings $11+1 = 100$ and $100+11 = 111$.

Prove that $L$ is not regular.

(10 marks)

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**Pumping lemma for regular languages**

Let $L$ be a regular language. Then there exists a constant $n \geq 1$ such that, for every string $w$ in $L$ with $|w| \geq n$, we can write $w = xyz$ in such a way that:

1. $|xy| \leq n$ (the initial section is not too long).
2. $y \neq \epsilon$ (the string to pump is not empty).
3. For all $k \geq 0$, the string $xy^kz$ is in $L$ (the string $y$ may be pumped any number of times, including 0, and the resulting string is still in $L$).