CIT3130 Theory of Computation

First mid-semester examination, September 2002

Time: 60 minutes
Perusal: 5 minutes
Marks: 40 marks

Please write neatly in ink and show all working.

Statements of the pumping lemma for regular languages is appended in case it is required.

1. Give a regular expression that defines the set of strings in $\{0, 1\}^*$ that start with 0 and do not contain 11. Example strings include 00 and 010010.

   (5 marks)

2. Construct a deterministic finite automaton that accepts the set of strings in $\{0, 1\}^*$ that do not contain 011. Example strings include 0, 1, 01, 010, 1101 and 0101001.

   (5 marks)

3. Transform the following nondeterministic finite automaton into an equivalent deterministic finite automaton. (Remember that a deterministic finite automaton must have a transition for each input symbol from each state.)

   $\begin{array}{c|c|c}
   \rightarrow & a & b \\
   \hline
   p & \{p, q\} & \{p, t\} \\
   q & \{r\} & \emptyset \\
   r & \{s\} & \emptyset \\
   * & \{s\} & \{s\} \\
   t & \emptyset & \{s\}
   \end{array}$

   (Note that $s$ is the only final state.)

   (10 marks)

4. Transform the following deterministic finite automaton $A$ into an equivalent regular expression $E$ (i.e., so that $E$ defines the same language that $A$ accepts).

   $\begin{array}{c|c|c}
   \rightarrow & a & b \\
   \hline
   p & q & q \\
   * & q & r \\
   r & q & p
   \end{array}$

   (Note that $q$ is the only final state.)

   (10 marks)

5. Prove that the language of balanced parenthesis strings ($\epsilon, (), ((())), \ldots$) is not regular.

   (10 marks)
Pumping lemma for regular languages

Let $L$ be a regular language. Then there exists a constant $n \geq 1$ such that, for every string $w$ in $L$ with $|w| \geq n$, we can write $w = xyz$ in such a way that:

1. $|xy| \leq n$ (the initial section is not too long).
2. $y \neq \epsilon$ (the string to pump is not empty).
3. For all $k \geq 0$, the string $xy^kz$ is in $L$ (the string $y$ may be pumped any number of times, including 0, and the resulting string is still in $L$).