Hierarchical Collections: Trees

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Outline

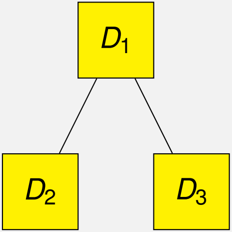
1. Trees
2. Expressions and Grammar Parsing
3. Search Trees

Hierarchical Collections

- Tree definition
- Types of Trees
- Binary Expressions
  - expression trees
  - tree traversals: pre-, in-, postorder
- Examples
  - generating Postfix
  - parsing

Tree Definition

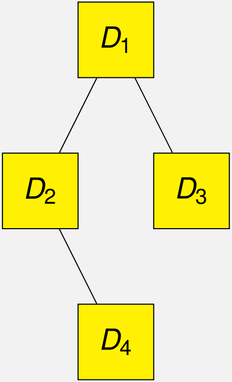
- Each node has at most one predecessor
  - Parent
- Many Successors
  - Children
- Siblings
  - nodes sharing the same parent (eg, D_2 and D_3)



Tree Definition (2)

- Topmost Node
  - root
- Childred, children of children, . . .
  - Descendants
  - Successors

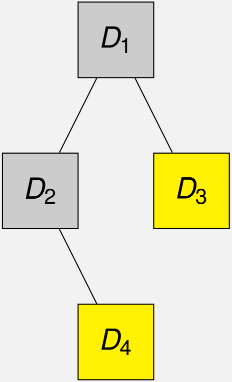
⇒ All nodes are successors of root



Tree Definition (3)

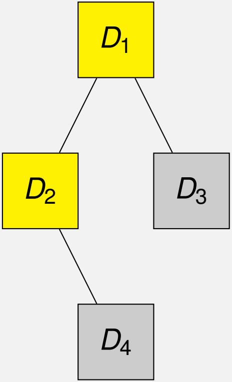
- Leaf Nodes
  - nodes without successors

  → D_3 and D_4
- Frontier
  - set of all leaf nodes



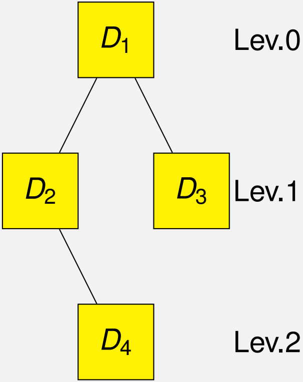
Tree Definition (4)

- Interior Nodes
  - nodes with at least one successor
  - → D_1 and D_2
- Ancestors
  - immediate or indirect predecessors
  - → D_1 is an ancestor of D_2, D_3, and D_4



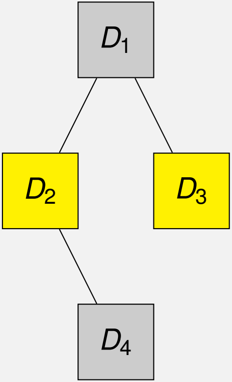
Tree Definition (5)

- Levels are numbered from 0
  - → level 0 is always the root
- This tree has *3 Levels*
  - Level 0: D_1
  - Level 1: D_2 and D_3
  - Level 2: D_4



Binary Trees

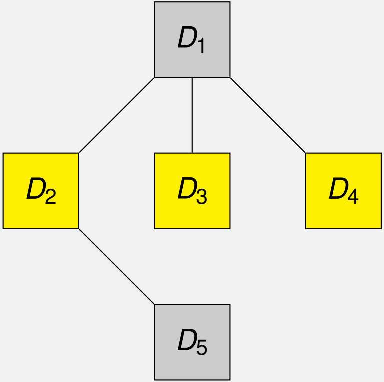
- Binary Trees
  → allow at most *two children* per node
- Generic Trees
  - allow any number of children per node



Generic Trees

- Order of the Tree
  - maximum *number of children* allowed for any given node

  → e.g. Order 3



Tree Applications

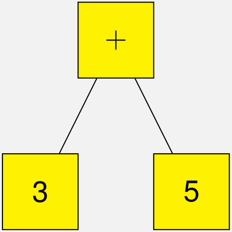
- Parsing Languages
  - Computer Languages, Mathematical Formulae
  - Natural Languages
- Searchable Data Structures
  - Databases (e.g., B-Trees)
  - Heaps and Balanced Trees
- Sorting and organising Data

Parsers

- Read in Expressions
  $\rightarrow$ $(2 + 3) * 5$
- Check Syntactical Correctness
  - is everything where it should be?
- Create Parse Tree
  - evaluator checks semantic meaning and processes the data in the Tree to produce meaningful output

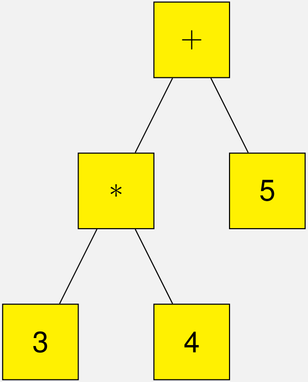
Binary Expressions

- Stored in Binary Trees
  $\rightarrow$ $3 + 5$
- Numbers
  - leaf nodes
- Operators
  - interior nodes
- Operands
  - contained in a subtree of the expression



Example Expression

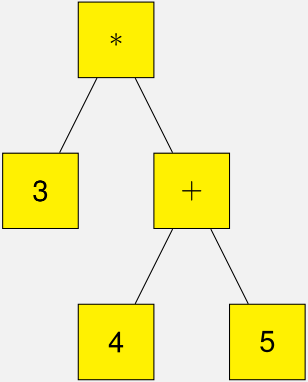
$3 * 4 + 5$



Operator Precedence

$$3 * (4 + 5)$$

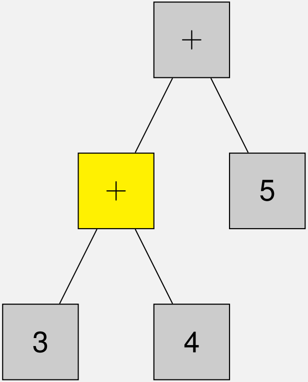
- The *higher* the precedence, the *lower* in the tree
  - $\rightarrow$ overridden by parentheses



Operator Precedence (2)

$3 + 4 + 5$

- if operators have equal precedence, the ones on the left appear lower in the tree when parsed from left to right!



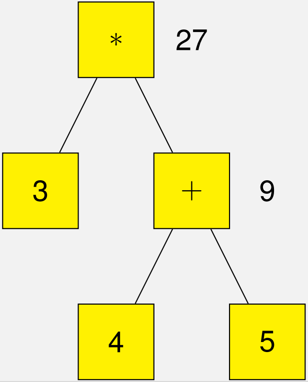
Evaluating an Expression Tree

- Begin at the root Node
- If a number, return it, otherwise
- Run the operator with the results of
  - evaluating its left and right subtrees, and
  - return this value

Evaluating Example

$3 * (4 + 5)$

- Evaluation starts at the top
- $*$ is an operator
  - $\Rightarrow$ evaluate left and right subtrees first!
- 3 is a number
  - $\Rightarrow$ return 3
- $+$ is an operator
  - $\Rightarrow$ evaluate left and right subtrees first!
- 4 is a number
  - $\Rightarrow$ return 4
- 5 is a number
  - $\Rightarrow$ return 5



Evaluation Pseudocode

Pseudo code for tree evaluation

```
evaluate(node)
{
   if node is a number
      return number;
   else
   {
      left = evaluate(node.left);
      right = evaluate(node.right);
      return compute(node, left, right);
   }
}
```

Binary Tree Traversals

- *Pre*order
  - $\rightarrow$ visit node, then go left, then go right
- *In*order
  - $\rightarrow$ go left, then visit node then go right
- *Post*order: Depth First
  - $\rightarrow$ go left, then go right, then visit node
- Breadth First
  - $\rightarrow$ level 0, then level 1, then level 2, etc.

Equivalence between Traversal and Notation

- Preorder, Inorder, and Postorder
  → correspond with Prefix, Infix, and Postfix notations of an expression
    - Infix: `3 + 5`
    - Prefix = Polish notation (PN): `+(3,5)`
    - Postfix = reverse Polish notation (RPN): `3 5 +`

⇒ use the same generic recursive algorithm!

Prefix Pseudocode

Prefix Evaluation

```
String prefix(node)
{
    if (node == NULL)
        return "";
    else
        return node +
               prefix(node.left) +
               prefix(node.right);
}
```

Infix Pseudocode

Infix Evaluation

```
String infix(node)
{
    if (node == NULL)
        return "";
    else
        return infix(node.left) +
               node +
               infix(node.right);
}
```

Postfix Pseudocode

Postfix Evaluation

```
String postfix(node)
{
    if (node == NULL)
        return "";
    else
        return postfix(node.left) +
               postfix(node.right) +
               node;
}
```

Grammar Parsing

Infix Expressions

Expression = Term { + | - Term }
Term = Factor { * | / Factor }
Factor = number | (Expression)

- Represents standard maths formulas
  - e.g.: $3 + 4 * (5 - (6/7))$
- can be used to create a parse tree!

  $\rightarrow$ recursive descent parsing

Recursive Descent Parsing

Expression = Term { + | - Term }

```
Expression()
{
  Term();
  while (token == '+' ||
         token == '-')
  {
      get_token();
      Term();
  }
}
```

Recursive Descent Parsing

Term = Factor { * | / Factor }

```
Term()
{
  Factor();
  while (token == '*' ||
         token == '/')
  {
      get_token();
      Factor();
  }
}
```

Recursive Descent Parsing

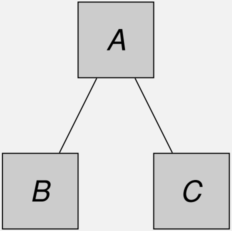
Factor = number | (Expression)

```
Factor() {
  switch (token) {
    case number:  get_token(); break;
    case '(':     get_token(); Expression();
        if (token != ')')
            error("No closing ')'");
        get_token();
    break;

    default:
        error("Error '%s'\n", token);
  }
}
```

Binary Search Tree

- “Sorted Array” stored in a tree
    - left to right order
    - e.g. **A B C**



Binary Tree Search

1. Start at the root node *n*
   - searching for an object *s*
2. if $s == n$ then we are finished
3. if $s < n$ then $n :=$ left child
4. if $s > n$ then $n :=$ right child
5. repeat from step 2 until finished
   - …either *s* has been found
   - …a leaf node has been reached, but *s* has not been found

Recursive Pseudocode

Recursive Pseudocode

```
search(s, node)
{
        if node == nil
                return nil;       // not in tree
        else if s == node->content
                return node;      // found
        else if s < node->content
                return search(s, node->left);
        else                       // s > node
                return search(s, node->right);
}
```

Search Tree Complexity

- Depends on the Balance of the Tree
- Unbalanced Tree:
  - $O(n)$
- Balanced Tree
  - $O(\log n)$
  - equivalent to Binary Search in Sorted Array

Balanced Trees

- Balanced Tree
  - Difference in height of both subtrees of any node in the tree is either 0 or 1
- Unbalanced Tree:
  - Difference of subtree heights > 1
- Perfectly Balanced Tree
  - Balanced Tree with leaves only on one or two levels

Creating a Search Tree

1. Incrementally
   - Sort in a new Node *n*
2. Search if *n* already exists
   - Finished if *n* exists (do nothing)
   - Otherwise add *n* as the left or right child of the last node searched (depending on whether *n* was smaller or bigger than the last node)
3. Produces an ad-hoc Search Tree
   - Not guaranteed to be balanced!

Balancing a Complete Tree

1. Write out the Search Tree in sorted order
    - e.g. in alphabetical order

    $\rightarrow$ write to sorted array/list

    $\rightarrow$ write to file
2. Read back the sorted data, creating a Balanced Tree
    - Recursively create Left Children, Root, then Right Children for each subtree
    - Creates a *perfectly balanced tree*!

Balancing ReadTree Algorithm

Balancing ReadTree Algorithm

```
BTNode *readTree(BufferedReader *file, int n)
{
        if (n <= 0) return nil;

        BTNode *node = [BTNode new];
        [node setLeft:  readTree(file, n/2)];
        [node setValue:  [file readLine]];
        [node setRight:  readTree(file,
(n-1)/2)];

        return node;
}
```

Self-Balancing Trees

- Problem: writing out and reading back
  - → takes time
  - → requires space
  - Read back the sorted data, creating a Balanced Tree
    - Sorted data are available in 3 places (original tree, file/array, and final, balanced tree)
- Alternative: keep the tree balanced
  - insertion operation needs to check if tree is still balanced
  - re-balance if adding a node breaks balance

Red-Black Tree

1. Every node is either red or *black*
2. The root node is *black*
3. All leaves are *black*
   - leaves are dummy empty nodes at the end of the tree
4. Both children of red nodes are *black*
5. All paths from any given node to its descendant leaves contain the *same number* of *black* nodes

Red-Black Tree Definitions

- Grandparent
  - the parent of the parent node
- Uncle
  - the “other child” of the grandparent, i.e.
    - `if (parent == grandparent.left)`
      `uncle = grandparent.right)`
    - `else` *// if (parent != grandparent.left)*
      uncle = grandparent.left)
- Both children of red nodes are *black*
- All paths from any given node to its descendant leaves contain the *same number* of *black* nodes

Red-Black Tree Insertion

- Add node as in a binary search tree
  - $\rightarrow$ default colour is red
- *Case 1:* new node n is root
  - $\rightarrow$ repaint as *black*
- *Case 2:* parent p of n is *black*
  - $\Rightarrow$ everything is fine!

Red-Black Tree Insertion

- *Case 3:* both parent and uncle are red

  $\rightarrow$ repaint parent and uncle as *black*
  $\rightarrow$ repaint grandparent as *red* (property 5)

  - may now violate property 2 (root is *black*) or property 4 (both children of red nodes are *black*)
    $\Rightarrow$ therefore recursively restart with case 1 on the grandparent

Red-Black Tree Insertion

- *Case 4:* parent *p* of new node *n* is red, uncle *u* is *black*
  - grandparent *g*
  - `if n == p.right && p == g.left`

    → perform *left rotation* to switch roles of *n* and *p*
  - `if n == p.left && p == g.right`

    → perform *right rotation* to switch roles of *n* and *p*

  → continue with *Case 5*!

Red-Black Tree Insertion

- *Case 5:* parent *p* of new node *n* is red, uncle *u* is *black*
  - switch the colours of *p* and grandparent *g*
  - `if n == p.left && p == g.left`
    $\rightarrow$ perform *right rotation* on *g*
  - `if n == p.right && p == g.right`
    $\rightarrow$ perform *left rotation* on *g*

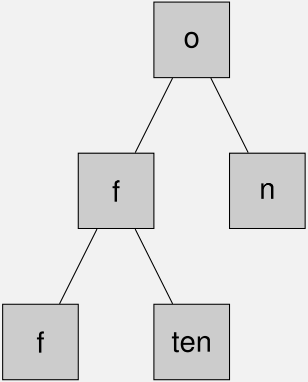
$\Rightarrow$ Terminal manoeuvre, no further repaint needed!

Strings in Search Trees

- Storing long strings in binary search trees can be inefficient
  - Requires full string (key) comparisons for every node
  - → $O(n \log n)$ search complexity if average string length approximates the number of nodes n
- Trie
  - Re*trie*val of keys while traversing a search tree

Tries

- trees that store the individual characters of the key strings
- common prefixes share the same path through the search tree, e.g.
  - on
  - off
  - often



Trie Efficiency

- Time and Space efficiency
  - $\rightarrow$ large number of long words
- Efficient for spell checking
  - $\rightarrow$ common prefixes determine tree height
  - English words do not share long common prefixes
    - 5-7 node visits, regardless of whether 10,000 or 100,000 words are stored!
    - compare with $13 = \log_2 10000$ or $17 = \log_2 100000$ node visits for optimal binary search trees!

Trie Challenges

- Prefix detection
  - how to distinguish words such as “are” and “area”
  - $\rightarrow$ requires a separate *end of word* mark
- Efficient search requires $O(1)$ character search in nodes
  - $\rightarrow$ requires (array) space for each node, indexed by char
    - 26+1 pointers for A-Z (plus end of word mark)
    - 127+1 pointers for ASCII
    - 65536 pointers for UTF-16
    - 4294967296 pointers for UTF-32 (full Unicode)
- Suffixes are different node types
  - $\rightarrow$ makes trie handling code more complex