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Voronoi-based Zoning Design by Multi-Objective Genetic Optimization

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***Abstract* — This paper presents a new approach to optimal zoning design. The approach uses a multi-objective genetic algorithm to define, in a unique process, the optimal number of zones of the zoning method along with the optimal zones, defined through Voronoi diagrams. The experimental tests, carried out in the field of handwritten digit recognition, show the superiority of new approach with respect to traditional dynamic approaches for zoning design, based on single-objective optimization techniques.**

Keywords: Genetic Algorithms, Handwritten Digit Recognition, Feature Extraction, NSGA II, Zoning Topology, Voronoi Diagrams.

I. INTRODUCTION

Zoning is one of the most successful feature extraction technique for handwritten digit recognition, since it is able to handle effectively handwritten pattern variability. In general, given a pattern image B, a zoning $Z_M=\{z_1, z_2, ..., z_M\}$ of B can be considered as a partition of B into M sub-images, named zones, each one providing information related to a specific part of the pattern [1].

When zoning is used, the definition of the most profitable zoning topology for the specific application is very important [2]. Traditional approaches use static zoning methods, in which zoning design is obtained by standard grids that are superimposed on pattern images. In this case, no a-priori information on feature distribution is used for defining the zoning method. Dynamic zoning techniques have been also proposed, in which zoning design is considered as an optimization problem and the optimal zoning method is found as the zoning which maximizes the classification performance, estimated by a well-defined cost function associated to the classification task [3]. For this purpose, Voronoi Diagrams have been proposed for zoning description, since they provide, given a set of M distinct points (named *Voronoi points*) in continuous space, a means of partitioning the space into M sub-regions (named *zones*) according to proximity relationships among the set of points [4]. Unfortunately, although dynamic zoning methods have been largely considered, little attention has been devoted so far to the automatic definition of the optimal number of zones for a given classification task. In other words Voronoi-based dynamic approaches proposed so far in literature, that are based on single-objective optimization techniques, assume the number of zones (M) to be fixed a-priori.

Starting from this consideration, this paper presents a new approach to Voronoi-based zoning design that combines, in a unique optimization process, the selection of the optimal number of zones along with the optimal Voronoi zones for a given classification problem. For this purpose, a multi-objective optimization problem is defined and a Non-dominated Sorting Genetic Algorithm (NSGA II) is considered for finding the optimal zoning method.

The experimental tests have been carried out in the field of handwritten digit recognition, using datasets extracted form the CEDAR database. The result shows that the optimal zoning methods derived from multi-objective optimization technique generally outperform traditional zoning methods based on single-objective optimization techniques.

The paper is organized as follows. Section II presents a brief overview of zoning methods. The problem of zoning-based classification is focused in Section III. Section IV presents the new approach to zoning design, based on multi-objective optimization. Section V shows the experimental results, carried out on handwritten digits of the CEDAR database. The conclusion of the paper is reported in Section VI.

II. ZONING METHODS: AN OVERVIEW

Traditional zoning methods use static zonings based on simple grids that are superimposed on the pattern image, as described in the survey papers of Mori et al [5].

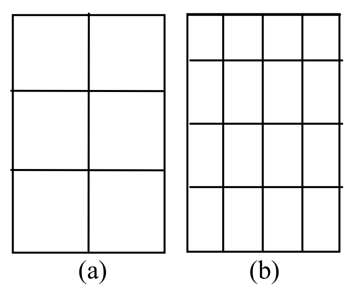

Fig. 1. Examples of uniform methods.

Some approaches use uniform u×v regular grids, determining partitions of the pattern image into regions of equal size. For example, Figure 1 shows two zoning methods based on regular grids 3×2 (Fig.1a) and 4×4 (Fig.1b). More precisely. Suen et al. [2, 6] present a hierarchical model to evaluate the distinctive parts of hand-printed characters. In particular, they use 2×2, 3×2, 1×2 and 2×1 grids. Bokser [7] uses a 3×3 regular grid for zoning design and computes the percentage of black pixels in each zone for classification. The same grid is used by Baptista and Kulkarni [8] and Impedovo et al. [9],

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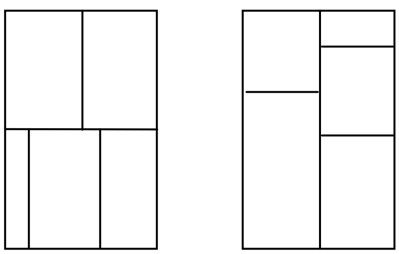
which extract geometrical feature distribution from each zone. Also Cao et al. [10] use a 3×3 grid for zoning. They also observe that when the contour curve is close to zone borders, small variations in the contour curve can lead to large variations in the extracted features. Therefore, they try to compensate for this by using a fuzzy border. Features detected near the zone borders are given fuzzy membership values to two or four zones. Oliveira et al. [11] adopt a 3×2 grid and extract contour-based features from each zone. Kimura and Shridhar [12] use a 4×4 regular grid for zoning design. They use the zoning to detect information from contour profiles of the patterns. In each zone the number of segments on the contour of the pattern with the same orientation is counted. Four basic orientations are considered: 0°,90°,+45°,-45°. The same 4×4 regular grid is used by Cha et al. [13] to extract gradient, structural and concavity information from the pattern image, and by Negi et al. [14] to derive the density of pixels in the different zones. Other approaches consider non uniform grids. For instance, Takahaski [15] uses vertical, horizontal and diagonal slices as zones. For each zone he determines the orientation histograms detected from pattern contours. Vertical zoning is obtained by a 1×4 grid, horizontal zoning is obtained by a 6×1 grid and two oriented (-45°, +45°) 6×1 grids are used for diagonal zonings.

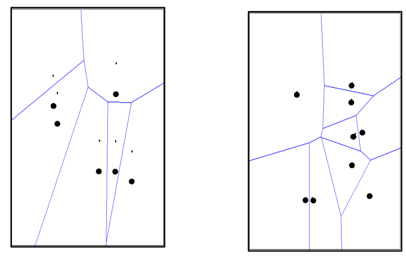


(a) (b)

Fig. 2. Examples of non-uniform methods.

Other researchers, use non regular grid for zoning design, resulting in non-uniformly splitting of the pattern image. For example, Figure 2 shows two different examples of non-uniform zonings. In both cases the pattern image is partitioned into five zones, according to a non-symmetrical strategy. Freitas et al. [16] consider concavity/convexity features, derived by the analysis of background pixels of the input image. Their approach uses the confusion matrices to make the zoning design process less empirical.

More recently, the zoning design problem as been considered as an optimization problem, which depends on the spatial distribution of features in the pattern image. The system of Valveny and Lopez [17] divide the pattern image into five rows and three columns. The size of each row and column is determined according to the discriminating capabilities of the diverse regions of the image. In the work of Dimauro et al. [18] zoning design is performed according to the analysis of discriminating capability of each zone, estimated by statistical parameters. In this case a region-growing process is proposed for zoning design. Di Lecce et al. [19] designed the zoning problem as an optimization problem in which the Shannon entropy is considered used to evaluate the discrimination capability of each zone, when a specific feature set is considered. More recently, zoning has been designed also by considering the specific requirements of the classification problem. Impedovo et al. [20, 21] define the optimal zoning as the zoning for which the Cost Function CF associated to the classification is minimum.



(a) (b)

Fig. 3. Examples of Voronoi-based methods.

In addition, Voronoi Tessellation is proposed for zoning description, since it provides, given a set of points (named Voronoi points) in continuous space, a means of naturally partitioning the space into zones, according to proximity relationships among the set of points. For example, Figure 3 shows two Voronoi-based zoning methods of 6 (Fig.1a) and 9 (Fig.1b) zones, respectively. In addition, changing the position of the Voronoi points corresponds to the modification of the zoning method. Therefore zoning description by Voronoi Tessellation offers the possibility to easily adapt the zoning to the specific characteristics of the classification problem. Impedovo et al. [4, 21] also propose a single-objective genetic algorithm for zoning design, in which each individual of the genetic population is a set of Voronoi points (corresponding to a zoning method) and the cost function associated to the classification is considered as fitness function. In addition, the role of membership functions for zoning based classification is also analyzed [21, 22, 23, 24]. Radtke et al. [25] present an automatic approach to define the zoning using Multi-Objective evolutionary algorithms. The idea is to provide a self adaptive methodology to define the zoning method according to two diverse optimality criteria: a minimal number of non-overlapping zones and an error rate as low as possible. Gagné and Parizeau [26] use a hierarchical zoning for handwritten character classification. They present a genetic programming based approach for optimizing the feature extraction step of a handwritten character recognizer. Their recognizer uses a multilayer perceptron as a classifier and operates on a hierarchical feature space of orientation, curvature, and center of mass primitives. The nodes of the hierarchy represent rectangular zones of their parent node whereas the tree root corresponds to the entire image pattern.

III. MULTI-OBJECTIVE OPTIMIZATION FOR ZONING DESIGN

In this paper the problem of optimal zoning design is considered as the result of a multi-objective optimization problem. More precisely, it is formulated as the problem to define the optimal zoning for which the cost function associated to the classification is minimum and in which the number of zones is minimum. Therefore, the two cost functions to be minimized are the following:

1) $CF_1(Z_M) = \mu \cdot Err(Z_M) + Rej(Z_M)$ (1a)

where:

- $Err(Z_M)$ is the error rate (estimated on the learning set);
- $Rej(Z_M)$ is the rejection rate (estimated on the learning set);
- the coefficient μ is the cost value associated to the treatment of an error with respect to a rejection.

2) $CF_2(Z_M) = M$ (1b)

where:

- M is the number of zones of the zoning method Z_M.

Of course, since Voronoi Diagram is used for zoning description, the problem of optimal zoning design becomes:

Find the set of Voronoi points $\{p^*_1, p^*_2, ..., p^*_M\}$ with minimum cardinality (M minimum) and for which it results :

$$Z^*_M \angle Z_M \quad , \forall Z_M \neq Z^*_M \qquad (2)$$

with:

- $Z^*_M = \{z^*_1, z^*_2, \ldots, z^*_M\}$, z^*_j being the Voronoi region corresponding to p^*_j , $\forall j=1,2,\ldots,M$;
- $Z_M = \{z_1, z_2, \ldots, z_M\}$, z_j being the Voronoi region corresponding to p_j , $\forall j=1,2,\ldots,M$.

and where $\angle$ is a partial order relation that depends on both CF_1 and CF_2 and that is precisely defined in the following of this Section (see eq. (5)).

In order to solve this optimization problem the Non-dominated Sorting Genetic Algorithm (NSGA II) has been considered [27]. In this case, individuals of the genetic population are evaluated by non-dominance and by spatial distribution criteria in order to derive a set of non-dominated solutions evenly spaced (such set is known as the *Pareto-front*), which represents the best configurations for the two objectives being optimized. Therefore, in order to solve the optimization problem (1), the following non-dominated sorting genetic algorithm is adopted [27]:

1. *Generate $2*N_{pop}$ random individuals each one of M elements:*

   For** number_of_generation=1 **to** Max_number_of_generation **do

   2. *Determine non-dominant fronts and compute the crowding-distance between individuals in each front;*
   3. *From the set of $2*N_{pop}$ individuals, select the best N_{pop} individuals (parents) as follows: between two individuals with differing nondomination ranks, chose the solution with the lower (better) rank; otherwise, if both individuals belong to the same front, then chose the solution that is located in a lesser crowded region.*
   4. *Generate a new set of N_{pop} individuals (offsprings) ad follows:*
      - a. *Apply Zoning Elitism*
      - b. *Apply Crossover*
      - c. *Apply Mutation*
   5. *Join the sets of the parents and the offsprings*

   End for

6. *Chose the zoning method corresponding to the best individual of the last generated population*

In the following, a detailed description of each phase of the algorithm is reported.

1) In this phase the initial population

$$Pop = \{\Phi_1, \Phi_2, ..., \Phi_\iota, ... , \Phi_{2*N_{Pop}}\} \qquad (3)$$

for the genetic algorithm is created. Each individual is a vector

$$\Phi_i = \langle p_1, p_2, .., p_j, .., p_M \rangle \qquad (4)$$

where each element $p_j=(x_j,y_j)$ is a Voronoi point corresponding to the zone z_j of $Z_M=\{z_1, z_2, ..., z_M\}$.

2) In this phase the non-dominant fronts are determined and the crowding-distance between individuals in each front is computed. These measures are useful to characterize each solution and to select the best ones.

More precisely, according to the approach of ref. [28], the algorithm for fast nondominated sort, for each solution Φ first calculates two entities:

- n_Φ , that counts the number of solutions which dominate the solution p,
- S_Φ, a set of solutions that the solution Φ dominates.

All solutions in the first nondominated front will have their domination count as zero. Now, for each solution with $n_\Phi=0$, the algorithm considers each member (Γ) of the set S_Φ and reduce its domination count by one. In doing so, if for any member the domination count becomes zero, the algorithm puts it in a separate list Q. These members belong to the second nondominated front. Now, the above procedure is continued with each member of Q and the third front is identified. This process continues until all fronts are identified. The algorithm is reported in the following:

Algorithm: Fast-nondominated-sort

```
For each Φ∈ Pop
        SΦ=∅
        nΦ=0
        For each Γ∈ P
                if (Φ∠ Γ) then
                        SΦ=SΦ∪{ Γ }
                else if (Γ ∠ Φ) then
                        nΦ=nΦ+1
        if nΦ=0 then
                Φrank=1
                F1=F1∪{ Γ }
i=1
while Fi≠0
        Q=∅
        For each Φ∈ Fi
                For each Γ ∈ Fi
                nΦ=nΦ-1
                if nΦ=0 then
                        Γrank=i+1
                        Q=Q∪{ Γ }
        i=i+1
        Fi=Q
```

The crowding distance has been introduced as an estimator of the density of solutions surrounding a particular solution in the population The computation of the crowding distance requires sorting the population according to each objective function value in ascending order of magnitude. Thereafter, for each objective function, the boundary solutions (solutions with smallest and largest function values) are assigned an infinite distance value. All other intermediate solutions are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. This calculation is continued with other objective functions. The overall crowding-distance value is calculated as the sum of individual distance values corresponding to each objective. Of course, each objective function is normalized before calculating the crowding distance, in fact f_m^{min} and f_m^{max} are respectively the minimum and maximum value of the m-th cost function for the individuals of the population. In the following the algorithm is reported in detail.

Algorithm: Crowding-distance assignment (C)

l=card(Pop)
for each Φ_i set C(i)=0
    for each objective CF_m do
        Pop= Sort(Pop, m)
        C(1)=C(l)=∞
            for i=2 to (l-1)
            $C(i)=C(i)+[CF_m(i+1)-CF_m(i-1)]/(f_m^{max}-f_m^{min})$

3) In this phase the best N_{Pop} individuals are selected from the set of $2{*}N_{pop}$ individuals. Precisely, since every individual in the population has two attributes:

(I) nondomination rank (i_{rank})
(II) crowding distance ($i_{distance}$);

a partial order relation ($\angle$) can be defined as follows:

$$Z^a) \angle Z^b \text{ if } (Z^a_{rank} < Z^b_{rank}) \text{ or } ((Z^a_{rank} = Z^b_{rank}) \text{ and } (Z^a_{distance} > Z^b_{distance})) \qquad (5)$$

That is, between two solutions with differing nondomination ranks, the solution with the lower (better) rank should be preferred. Otherwise, if both solutions belong to the same front, then the best solution is that located in a lesser crowded region.

4) In this phase the new set of N_{Pop} individuals (offsprings) is generated. This is performed according to the following genetic operations [4]:

a) Zoning Elitism. The zoning elitist technique selects randomly some individuals of the population and removes the element corresponding to the less significant zone from the individual. The significance of an element (i.e. a zone) is here defined according to the number of instances a feature in the learning patterns lies in that zone. The lower the number of instances in a zone the lower the significance of that zone. This operation allows the production of zoning methods with a reduced number of zones. It is worth noting that this strategy does not apply to two-zone zoning methods.

b) Crossover. One-point crossover is used to combine information from diverse individuals. Let

$$\Phi_i = \left\langle p_1{}^a, p_2{}^a, .., p_j{}^a, .., p_{M_1}{}^a \right\rangle \qquad (6a)$$

and

$$\Phi_i = \left\langle p_1{}^b, p_2{}^b, .., p_j{}^b, .., p_{M_2}{}^b \right\rangle \qquad (6b)$$

be two individuals selected for crossover, the two offspring individuals

$$\Phi_i = \left\langle \not p_1{}^a, \not p_2{}^a, .., \not p_j{}^a, .., \not p_{M_2}{}^a \right\rangle \qquad (7a)$$

and

$$\Phi_i = \left\langle \not p_1{}^b, \not p_2{}^b, .., \not p_j{}^b, .., \not p_{M_1}{}^b \right\rangle \qquad (7b)$$

of the next generation are obtained as follows:

- $\not p^a_s = p^a_s$, for s=1,…j ; $\not p^a_s = p^b_s$, for s=j+1,…M2
- $\not p^b_s = p^b_s$, for s=1,…j ; $\not p^b_s = p^a_s$, for s=j+1,…M1

being s a random integer in the range $[1, \min(M_1,M_2)]$.

c) Mutation. A non-uniform mutation operator has been used. Let us consider the individual Φ_ι and an element selected for mutation, according to a mutation probability Mut_prob. The non-uniform mutation changes p_j in the new element $\tilde{p}_j = (\tilde{x}_j, \tilde{y}_j)$ that is defined as follows:

$$\begin{cases} \tilde{x}_j = x_j + \delta \cdot \cos(\varphi) \\ \tilde{y}_j = y_j + \delta \cdot \sin(\varphi) \end{cases} \qquad (8)$$

where:

- φ is a random value generated according to a uniform distribution, $\varphi \in [0,2\pi[$;
- δ is a displacement determined according to the following equation:

$$\delta = \delta\_displ \cdot \left(1 - \nu^{\left[1 - \frac{iter}{N^{iter}}\right]^b} \right) \qquad (9)$$

being:

- ν a random value generated in the range [0, 1], according to a uniform distribution;
- *δ_displ* the maximum displacement allowed;
- *b* a parameter determining the degree of non-uniformity;
- *iter* the counter of the generations performed;
- N^{iter} the maximum number of generations.

It is worth noting that eq. (9) causes the operator to search the space almost uniformly initially, when *iter* is small, and locally in later stages [4].

5) In this phase the two sets of parents and offsprings are joint together.

Steps from (2) to (5) are repeated until *Max_number_of_generation* successive populations of individuals are generated.

6) In this phase the optimal zoning is obtained by the best individual of the last-generated population.

IV. Experimental Results

The experiments have been carried out using 18467 learning patterns (BR directory) and 2189 testing patterns (BS directory) of the CEDAR database. The feature set $F=\{f_1,...,f_9\}$ is considered for pattern description, where [4]: f_1 - *holes*; f_2 - *vertical-up cavities*; f_3 - *vertical-down cavities*; f_4 - *horizontal-right cavities*; f_5 - *horizontal-left cavities*; f_6 - *vertical-up end-points;* f_7 - *vertical-down end-points;* f_8 -

horizontal-right end-points; f_9 - horizontal-left end-points. The following parameter values have been considered for the genetic algorithm: N_{Pop}=10; *Max_number_of_generation* = 40; *Mut_prob* = 0.35; δ*_displ*=5; *b*=1.0; λ*_displ*=0.5, *c*=3.0.

Table I reports the performance obtained when single objective and multi-objective optimization techniques are considered. Using single-objective optimization techniques, optimal zoning methods with M=2,4,6,9,16 zones have been obtained. In this case the best result is for M=9, for which an error rate equal to 14% can be achieved. When the multi-objective optimization technique is considered the optimal number of zones is M=11 and an error rate equal to 6% has been registered.

Table I. Optimal Zoning Methods: Performances (μ=1)

Genetic Algorithm	M	Error Rate
Single-Objective	2	20%
	4	14%
	6	14%
	9	14%
	16	24%
Multi-Objective	11	6%

V. CONCLUSION

This paper addresses the problem of optimal zoning design, by using multi-objective genetic algorithms. Despite traditional approaches, the new strategy allows the definition , in a unique optimization process, of the zoning with optimal (minimum) number of zones and the best performances. The strategy, that is based on the (nondominant sorting genetic algorithm (NSGA II), has been applied to the field of handwritten digit recognition. The experimental results demonstrate the effectiveness of the approach with respect to traditional techniques based on single-objective genetic algorithms.

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