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Meta-Interpretive Learning of Logic Programs

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Motivation

Logic Programming [Kowalski, 1976]

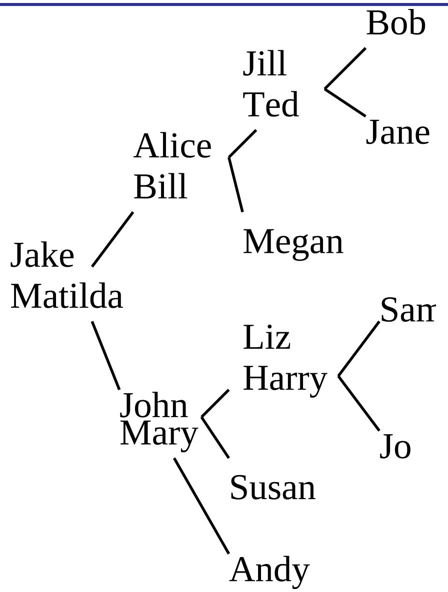
Inductive Logic Programming [Muggleton, 1991]

Machine Learn arbitrary programs

State-of-the-art ILP systems lacked Predicate Invention and Recursion [Muggleton et al, 2011]

Family relations (Dyadic)

Family tree



Target Theory

$$father(ted, bob) \leftarrow$$

$$father(ted, jane) \leftarrow$$

$$parent(X, Y) \leftarrow mother(X, Y)$$

$$parent(X, Y) \leftarrow father(X, Y)$$

$$ancestor(X, Y) \leftarrow parent(X, Y)$$

$$ancestor(X, Y) \leftarrow parent(X, Z), \\ ancestor(Z, Y)$$

Generalised Meta-Interpreter

$$prove([], BK, BK).$$

$$prove([Atom|As], BK, BK\_H) :-$$

$$\quad metarule(Name, MetaSub, (Atom \text{ :- } Body), Order),$$

$$\quad Order,$$

$$\quad save\_subst(metasub(Name, MetaSub), BK, BK\_C),$$

$$\quad prove(Body, BK\_C, BK\_Cs),$$

$$\quad prove(As, BK\_Cs, BK\_H).$$

Metarules

Name	Meta-Rule	Order
Instance	$P(X,Y) \leftarrow$	$True$
Base	$P(x,y) \leftarrow Q(x,y)$	$P \succ Q$
Chain	$P(x,y) \leftarrow Q(x,z), R(z,y)$	$P \succ Q, P \succ R$
TailRec	$P(x,y) \leftarrow Q(x,z), P(z,y)$	$P \succ Q,$ $x \succ z \succ y$

Meta-Interpretive Learning (MIL)

First-order	Meta-form
Examples	**Examples**
ancestor(jake,bob) ←	prove([ancestor(jake,bob),
ancestor(alice,jane) ←	ancestor(alice,jane)], ..) ←
Background Knowledge	**Background Knowledge**
father(jake,alice) ←	instance(father,jake,john) ←
mother(alice,ted) ←	instance(mother,alice,ted) ←
Instantiated Hypothesis	**Abduced facts**
father(ted,bob) ←	instance(father,ted,bob) ←
father(ted,jane) ←	instance(father,ted,jane) ←
p1(X,Y) ← father(X,Y)	base(p1,father) ←
p1(X,Y) ← mother(X,Y)	base(p1,mother) ←
ancestor(X,Y) ← p1(X,Y)	base(ancestor,p1) ←
ancestor(X,Y) ← p1(X,Z), ancestor(Z,Y)	tailrec(ancestor,p1,ancestor) ←

Minimising sets of Metarules [ILP 2014]

Set of Metarules	Reduced Set
$P(X,Y) \leftarrow Q(X,Y)$	
$P(X,Y) \leftarrow Q(Y,X)$	$P(X,Y) \leftarrow Q(Y,X)$
$P(X,Y) \leftarrow Q(X,Y), R(Y,X)$	
$P(X,Y) \leftarrow Q(X,Y), R(Y,Z)$	
$P(X,Y) \leftarrow Q(X,Y), R(Z,Y)$	
$P(X,Y) \leftarrow Q(X,Z), R(Z,Y)$	$P(X,Y) \leftarrow Q(X,Z), R(Z,Y)$
..	
$P(X,Y) \leftarrow Q(Z,Y), R(Z,X)$	

Expressivity of H_2^2

Given an infinite signature H_2^2 has Universal Turing Machine expressivity [Tarnlund, 1977].

utm(S,S)	$\leftarrow$	halt(S).
utm(S,T)	$\leftarrow$	execute(S,S1), utm(S1,T).
execute(S,T)	$\leftarrow$	instruction(S,F), F(S,T).

Q: How can we limit H_2^2 to avoid the halting problem?

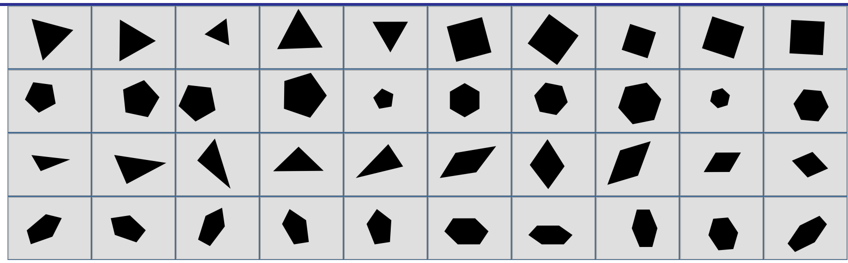
Metagol implementation (1)

- Ordered Herbrand Base [Knuth and Bendix, 1970; Yahya, Fernandez and Minker, 1994] - guarantees termination of derivations. Lexicographic + interval.
- Episodes - sequence of related learned concepts.
- $0, 1, 2, ..$ clause hypothesis classes tested progressively.
- Log-bounding (PAC result) - $log_2 n$ clause definition needs n examples.
- YAP implementation - https://github.com/metagol/metagol
.

Metagol implementation (2)

- Andrew Cropper's YAP implementation - https://github.com/metagol/metagol
  .
- Hank Conn's Web interface - https://github.com/metagol/metagol_web_interface
  .
- Live web-interface - http://metagol.doc.ic.ac.uk

Vision applications (1)

	
Staircase	Regular Geometric
ILP 2013	ILP 2015

stair(X,Y) :- stair1(X,Y).

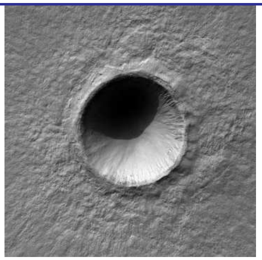
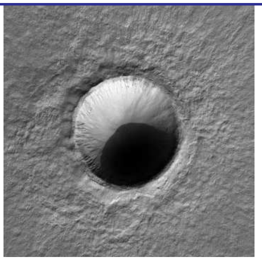
stair(X,Y) :- stair1(X,Z), **stair(Z,Y)**.

stair1(X,Y) :- vertical(X,Z), horizontal(Z,Y).

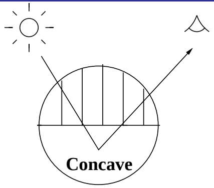
Learned in 0.08s on laptop from single image.

Note Predicate invention and **recursion**.

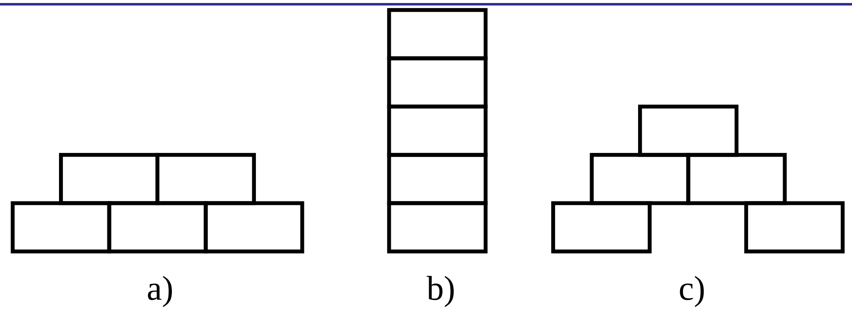
Vision applications (2) - ILP2017 - Object invention

Example Mars Images	  lit(obj1,north). lit(obj1,south).

Background Knowledge	light_path(X,X). light_path(X,Y) :- reflect(X,Z), light_path(Z,Y). highlight(X,Y):- contains(X,Y), brighter(Y,X), light(L), light_path(L,Y), reflector(Y), light(Y,O), observer(O). hl_angle(obj1,hlight,south). % highlight angle opposite(north,south). opposite(south,north).

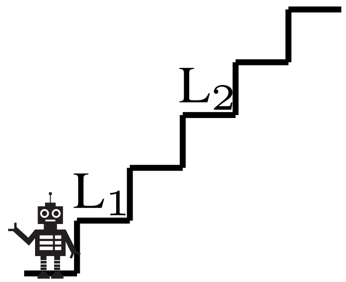
Hypothesis Image1	 lit(A,B):- lit1(A,C), lit3(A,B,C). lit1(A,B):- highlight(A,B), lit2(A), lit4(B). lit3(A,B,C):- hl_angle(A,B,D), opposite(D,C). lit2(obj1). % concave lit4(hlight). % highlight light(light1). observer(observer1). reflector(hlight). reflect(obj1,hlight). reflect(hlight, observer1).

Robotic applications



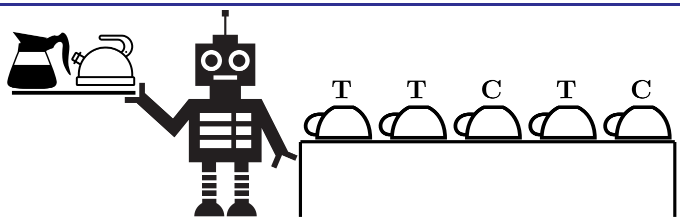
Building a Stable Wall

IJCAI 2013



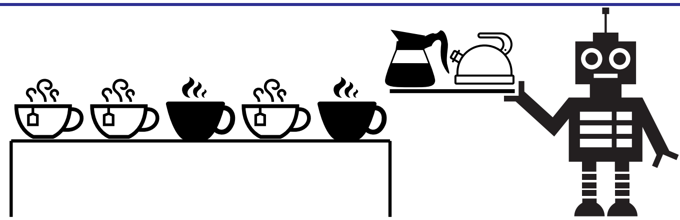
Learning Efficient Strategies

IJCAI 2015



Initial state

IJCAI 2016



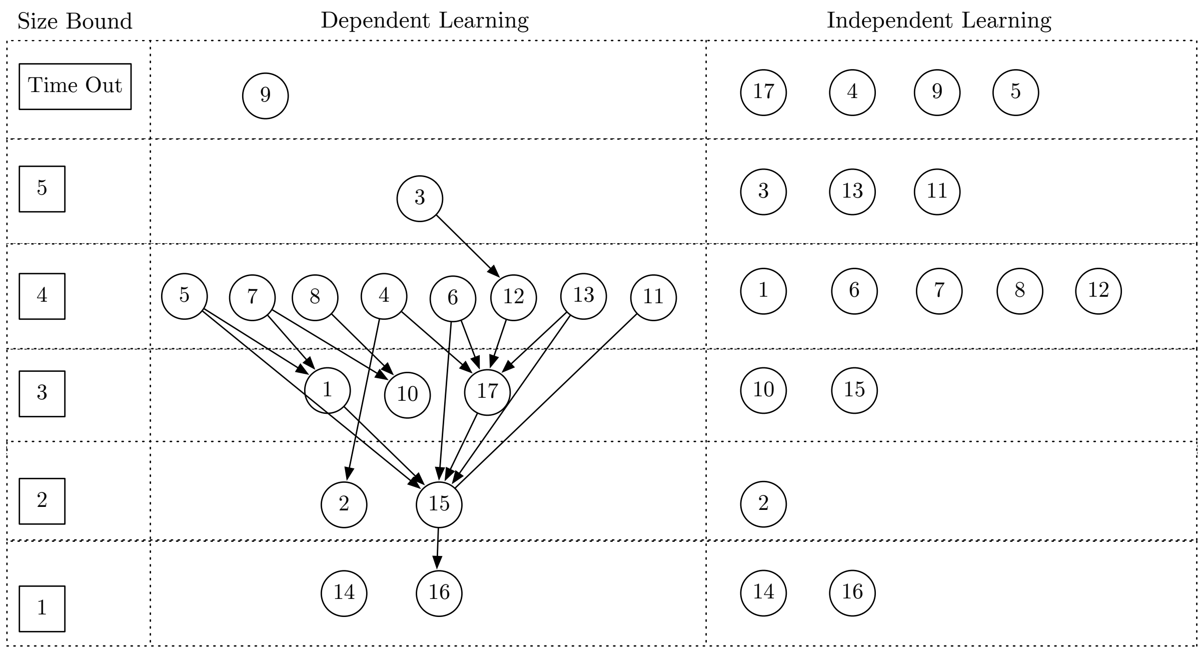
Final state

Abstraction and Invention

Language applications

Formal grammars [MLJ 2014]

Dependent string transformations [ECAI 2014]



Chain of programs from dependent learning

f_{03}(A,B) :- f_{12_1}(A,C), f_{12}(C,B).

f_{12}(A,B) :- f_{12_1}(A,C), f_{12_2}(C,B).

f_{12_1}(A,B) :- f_{12_2}(A,C), $skip1$(C,B).

f_{12_2}(A,B) :- f_{12_3}(A,C), $write1$(C,B,'.').

f_{12_3}(A,B) :- $copy1$(A,C), f_{17_1}(C,B).

f_{17}(A,B) :- f_{17_1}(A,C), f_{15}(C,B).

f_{17_1}(A,B) :- f_{15_1}(A,C), f_{17_1}(C,B).

f_{17_1}(A,B) :- $skipalphanum$(A,B).

f_{15}(A,B) :- f_{15_1}(A,C), f_{16}(C,B).

f_{15_1}(A,B) :- $skipalphanum$(A,C), $skip1$(C,B).

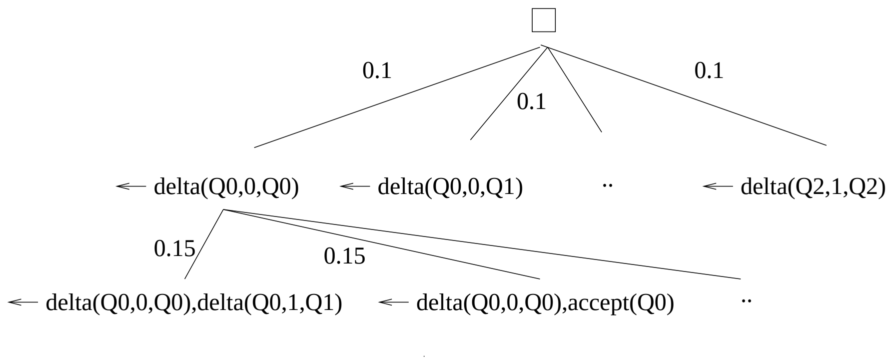
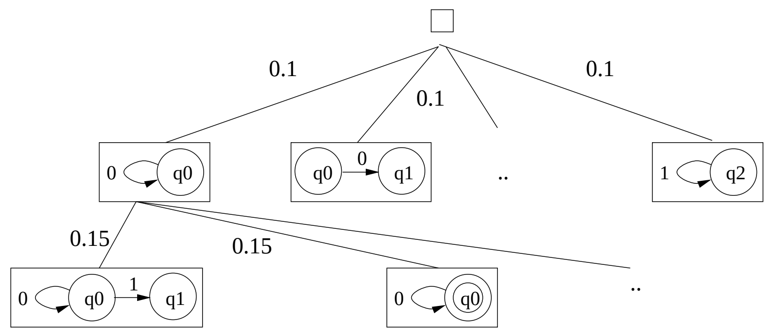
f_{16}(A,B) :- $copyalphanum$(A,C), $skiprest$(C,B).

Other applications

Learning proof tactics [ILP 2015]

Learning data transformations [ILP 2015]

Bayesian Meta-Interpretive Learning

Clauses	
Finite State Acceptors (FSAs)	

Related work

Predicate Invention. Early ILP [Muggleton and Buntine, 1988; Rouveirol and Puget, 1989; Stahl 1992]

Abductive Predicate Invention. Propositional Meta-level abduction [Inoue et al., 2010]

Meta-Interpretive Learning. Learning regular and context-free grammars [Muggleton et al, 2013]

Higher-order Logic Learning. Without background knowledge [Feng and Muggleton, 1992; Lloyd 2003]

Higher-order Datalog. HO-Progol learning [Pahlavi and Muggleton, 2012]

Conclusions and Challenges

- New form of Declarative Machine Learning [De Raedt, 2012]
- H^2_2 is tractable and Turing-complete fragment of High-order Logic
- Knuth-Bendix style ordering guarantees termination of queries
- Beyond classification learning - strategy learning

Challenges

- Generalise beyond Dyadic logic
- Deal with classification noise
- Active learning
- Efficient problem decomposition
- Meaningful invented names and types

Bibliography

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- D. Lin, E. Dechter, K. Ellis, J.B. Tenenbaum, S.H. Muggleton. Bias reformulation for one-shot function induction. ECAI 2014.